Collateral Reuse and Financial Stability

Jin-Wook Chang[1] Grace Chuan[2]

[1]Federal Reserve Board

[2]Columbia University

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George Washington University

Motivation

- Securities financing transactions (SFTs): use securities to borrow cash or vice versa (repo, reverse repo, securities lending, and etc.)

  As of March 2025:
  - Outstanding SFTs in EU $>$ **€15 trillion**
  - Outstanding repos/reverse repos in the US $>$ **$6 trillion**

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- Collateral posted by one side can be reused (rehypothecated) by the other side

  - Infante et al. (2020) find that the collateral multiplier (degree of collateral reuse) for U.S. Treasuries is around 10

- Policy makers have discussed potential systemic risks stemming from collateral reuse (Aitken and Singh, 2010; FSB, 2017)

- However, the industry has pushed back against this idea (Hill, 2014; ICMA 2025)

Research Question: What is the effect of collateral reuse on financial stability?

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    - BUT, all these factors confound the actual effect of collateral reuse

Research Question: What is the effect of collateral reuse on financial stability?

- Many papers on collateral reuse with leverage, liquidity, safe asset demand, risks of lender default, and collateral runs
  - BUT, all these factors confound the actual effect of collateral reuse
- **Isolated effects** of collateral reuse on financial stability are unclear:
  - Holding counterparty liabilities fixed, greater collateral reuse can guarantee and protect more debt obligations with fewer assets
  - More debt obligations depend on the same collateral, whose value can potentially drop
  - More collateral reuse can incentivize agents to take on greater risk
- Main contribution: new model to isolate effects of collateral reuse

Relation to the Literature 1/2

- Contagion in financial networks:
  Allen and Gale (2000); Eisenberg and Noe (2001); Elliott, Golub, and Jackson (2014); Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015); Donaldson et al. (2022); **Chang and Chuan (2024)**

- Collateral reuse:
  Infante (2019); Gottardi, Maurin, and Monnet (2019); Park and Kahn (2019); Infante and Vardoulakis (2021); Chang (2021); Luu et al. (2021); Maurin (2022); Brumm et al. (2023); Infante and Saravay (2024)

- Multiple equilibria in financial networks:
  Rogers and Veraart (2013); Roukny, Battiston, and Stiglitz (2018); Bernard, Capponi, and Stiglitz (2022); Capponi, Corell, and Stiglitz (2022); Jackson and Pernoud (2024)

Relation to the Literature 2/2

- Equilibrium selection:
  - Global games:
    Carlsson and Van Damme (1993); Morris and Shin (1998); Bernardo and Welch (2004); Goldstein and Pauzner (2005); Kuong (2021); Kashyap, Tsomocos, and Vardoulakis (2024); Eisenbach and Phelan (2025)
  - Best response dynamics:
    Gilboa and Matsui (1991); Matsui (1992); Mäder (2024)
- Systemic risk and excessive risk-taking:
  Castiglionesi, Feriozzi, and Lorenzoni (2019); Elliott, Georg, and Hazell (2021); Altinoglu and Stiglitz (2023); Jackson and Pernoud (2024); Shu (2024)

Preview of Main Results

1. There are three equilibria:
   (1) best (maximum) equilibrium
   (2) intermediate equilibrium
   (3) worst (minimum) equilibrium

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Collateral reuse $\uparrow \Rightarrow$ Crisis is less likely but more severe

Implications

- Collateral reuse alone is not a concern for financial stability

- Collateral reuse can still negatively impact financial stability through indirect effects on risk-taking decisions

- Supporting the price and liquidity of collateral assets can significantly improve social surplus
  (dealer of last resort, SRF, and etc.)

Model Overview

- Based on Chang and Chuan (2024), which extends Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015)
- Key features:
  1. **Assets** can be used as collateral of a debt contract
  2. Asset price is endogenously determined
- Main objects of interest: network structure and collateral amount

Agents, Goods, and Long-term Project

- Three periods: $t = 0, 1, 2$
- Two goods in economy: cash (e) and an asset (h)
  - Cash is the only consumption good and is storable
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  - Long-term investment project gives ξ amount of cash at $t = 2$
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    $\Longrightarrow$ Early liquidation = inefficiency
- State of the liquidity shocks: $\omega \equiv (\omega_1, \omega_2, \ldots, \omega_n) \in \Omega$
  $\Longrightarrow$ Liquidity shock ϵ = **senior liability**

Collateralized Debt Contracts and Constraints

- Debt contract btw lender i and borrower j from $t = 0$ to $t = 1$
- Debt amount d_{ij} and collateral(-to-debt) ratio c_{ij}
- Lender has full recourse and collateral transfer is frictionless

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- Debt amount d_{ij} and collateral(-to-debt) ratio c_{ij}
- Lender has full recourse and collateral transfer is frictionless
- Collateral constraints (allow reuse of collateral):

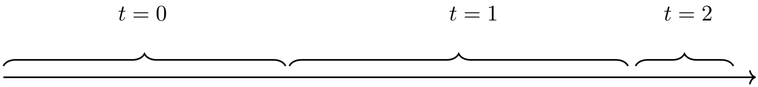
$$\sum_{k \in N} c_{jk} d_{jk} + h_j \geq \sum_{i \in N} c_{ij} d_{ij} \qquad \forall j \in N$$

- Resource constraints:

$$\sum_{i \in N} h_i \geq \sum_{i \in N} c_{ij} d_{ij} \qquad \forall j \in N$$

- Collateralized debt network: $(C, D) \equiv \left([c_{ij}]_{i,j \in N}, [d_{ij}]_{i,j \in N} \right)$

Timeline



Given:

1. (C, D) debt network
2. $(e_j, h_j)_{j \in N}$ cash and asset holdings
3. ξ investment project

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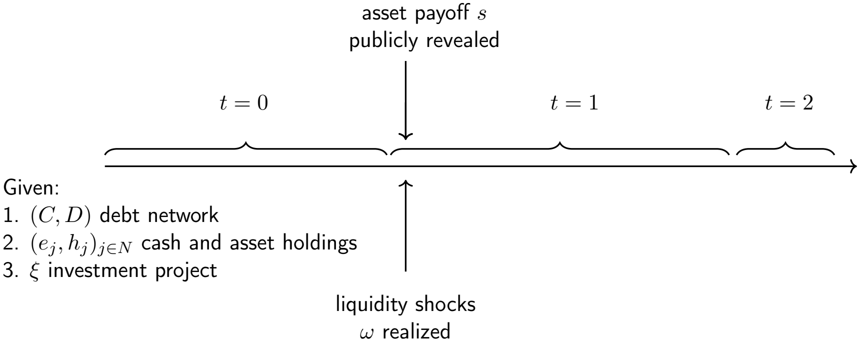
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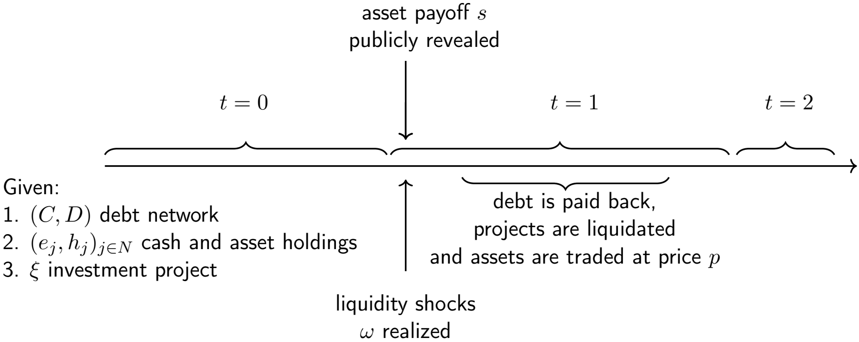
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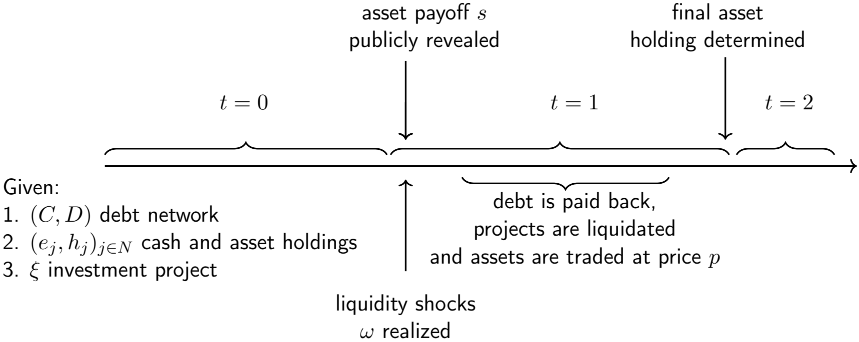

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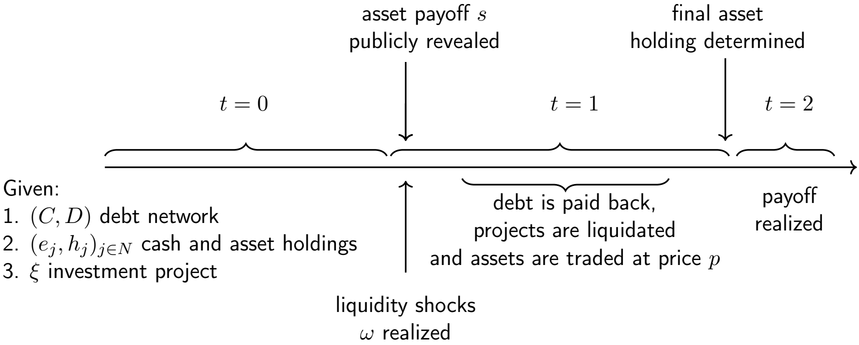
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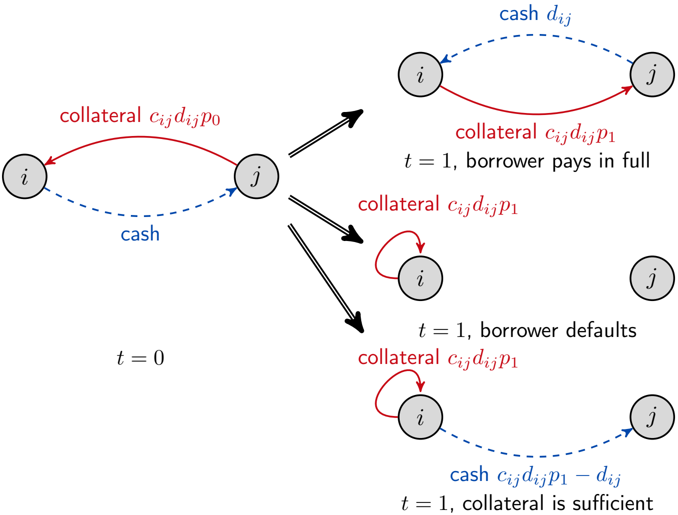
Flows of Cash and Collateral



Payment Rule

- j's total cash inflow:

$$a_j(p) \equiv e_j + h_j p + \sum_{k \in N} c_{jk} d_{jk} p - \sum_{i \in N} c_{ij} d_{ij} p + \sum_{k \in N} \underbrace{x_{jk}(p)}_{\substack{\text{actual payment} \\ \text{to } j \text{ from } k}}$$

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- j's total cash outflow:

$$b_j(p, \omega) \equiv \sum_{i \in N} \underbrace{(d_{ij} - c_{ij} d_{ij} p)}_{\text{net debt amount}} + \omega_j \epsilon,$$

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- Net payment to agent i from agent j at asset price p at $t = 1$:

$$x_{ij}(p) = \min \left\{ d_{ij} - c_{ij} d_{ij} p \;, \quad q_{ij}(p) \left[a_j(p) + \sum_{i \in N} [c_{ij} d_{ij} p - d_{ij}]^+ - \omega_j \epsilon \right]^+ \right\}$$

$$\text{pro rata weights: } q_{ij}(p) = \frac{[d_{ij} - c_{ij} d_{ij} p]^+}{\sum_{k \in N} [d_{kj} - c_{kj} d_{kj} p]^+}$$

Agent Wealth

- Net wealth of agent j:

$$m_j(p) \equiv a_j(p) - b_j(p)$$
$$= e_j + h_j p + \sum_{k \in N} c_{jk} d_{jk} p - \omega_j \epsilon - \sum_{i \in N} d_{ij} + \sum_{k \in N} x_{jk}(p)$$

- If $m_j(p) < 0$, then agent j defaults, i.e., $j \in \mathcal{D}(p)$

- **Excess net wealth can be used to purchase assets**

Fire-Sales

- Assume that agents sell up to its asset holdings amount
- *Fire-sale* amount of agent j:

$$\phi_j(p) = \min\left\{[h_j p - m_j(p)]^+, h_j p\right\}$$

Agent's optimization problem

- If agent j's net cash flow, $m_j(p) - h_j p$, covers payments
  $\Longrightarrow \phi_j(p) = 0$
- If agent j's net cash flow + sale of assets covers payments
  $\Longrightarrow 0 < \phi_j(p) < h_j p$
- If agent j's net cash flow + sale of assets does not cover payments
  $\Longrightarrow \phi_j(p) = h_j p$

Market Clearing Condition

- Asset's fundamental value s is common knowledge, and $p \leq s$.
- Liquidity constrained price $p < s$, i.e., cash-in-the-market pricing
- Price of collateral (p) is determined by the mkt clearing cdn:

$$\sum_{j \in N} [m_j(p) - h_j p]^+ = \sum_{i \in N} \phi_i(p) \qquad \text{if } 0 \leq p < s$$

$$\sum_{j \in N} [m_j(s) - h_j s]^+ \geq \sum_{i \in N} \phi_i(s) \qquad \text{iff } \quad p = s,$$

- Excess net wealth vs fire sale amount

Market Clearing Implications

- **Lemma.** If aggregate positive net wealth $\sum_{j \in N} [m_j(p)]^+ > 0$, then it is strictly increasing in the asset price p.

- **Lemma.** The market clearing asset price can be represented as

$$p = \min\left\{ \frac{\sum_{j \in N} [m_j(p)]^+}{\sum_{j \in N} h_j}, s \right\}$$

Full Equilibrium

Definition

For given $(N, C, D, e, h, s, \omega)$*, if payments* $\{x_{ij}(p)\}$ *satisfy the payment rule ,* $\{m_j(p)\}$ *is determined by net wealth equation, the fire-sale amount* $\{\phi_j(p)\}$ *is determined by fire-sale equation, and price* p *clears the market, then* $(\{x_{ij}\}, \{m_j\}, \{\phi_j\}, p)$ *is a* ***full equilibrium****.*

Proposition (Chang and Chuan (2024))

For any given collateralized debt network, cash and asset holdings, asset payoff, and realization of shocks $(N, C, D, e, h, s, \omega)$*, a full equilibrium always exists and is generically unique for a given equilibrium price. Furthermore, there exists a full equilibrium with the highest price among the* ***set of full equilibria****.*

Contagion and Social Surplus

- For any given collateralized debt network and full equilibrium, the utilitarian social surplus in the economy at $t = 2$ is:

$$U = \sum_{i \in N} (\pi_i + T_i),$$

where $T_i \leq \epsilon$ is the transfer from agent i to senior creditors (liquidity shock) and π_i is agent i's profit at $t = 2$

Lemma

For any full equilibrium, the social surplus in the economy is equal to

$$U = \sum_{j \in N} (e_j + h_j s + \xi) - \sum_{i \in N} l_i$$

Regularity Assumptions

- Regular networks: $\sum_{i \in N} d_{ij} = \sum_{i \in N} d_{ji} = d$ for all $j \in N$
- Homogeneous endowments: $e_i = e_0$ and $h_i = h_0$ for all $i \in N$
- Uniform collateral ratio: $c_{ij} = c$ for all $i, j \in N$
  - Collateral ratio c is equivalent to the *degree of collateral reuse*
- Liquidity shock: $\epsilon > ne_0$ received by one agent

Collateral Ratio and Connectedness Thresholds

Proposition (Chang and Chuan (2024))

1. *If* $c \geq \bar{c}(s,n) \equiv \frac{1}{s}$*, then no agent defaults in the maximum equilibrium for any given network* D*.*
2. *If* $c \geq \underline{c}(s,n) \equiv \frac{d-(n-1)e_0+h_0 s}{ds}$*, then the asset price is* $p = s$ *in the maximum equilibrium for any given network* D*.*

- Focus on cases in which collateral can play its role in mitigating contagion:

$$\underline{c}(s,n) \leq c$$

- Assume that agents are *sufficiently interconnected*

Details

Main Result 1: Three Unique Equilibria

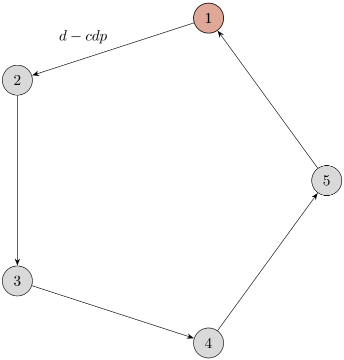
Proposition

For any network D and any $c \geq \underline{c}(s, n)$, three different equilibria exist generically:

1. *Maximum equilibrium: $p = s$ with the least number of defaults*
2. *Minimum equilibrium: $p = 0$ with all agents default*
3. *Intermediate equilibrium: $0 < p < s$ s.t. market clearing condition implies cash-in-the-market pricing, i.e., $p = \frac{\sum_{j \in N} [m_j(p)]^+}{nh_0}$*

- Significant swings in social surplus based on coordination of eqm
- Interventions can prevent bad equilibrium (dealer of last resort)

Example: Maximum Equilibrium



- Each agent has
  $e_0 = 2$: 2 units of cash
  $h_0 = 2$: 2 units of assets
  $d = 10$: debt amount
  $c = 1$: collateral ratio
- Asset's fair value $s = 1$
- Agent 1 hit by liquidity shock

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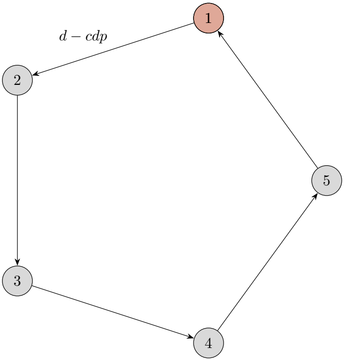

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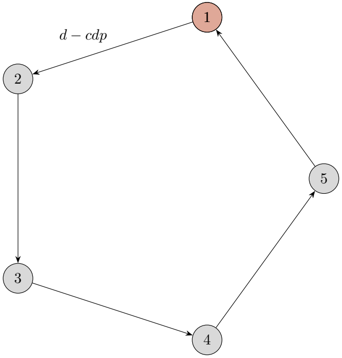
- Agent 1 hit by liquidity shock
- If $p = 1$, net debt amount is $d - cdp = 10 - 1 \times 10 \times 1 = 0$, and no defaults
- Surviving agents' aggregate cash 8 can clear the 2 assets on sale at its fundamental value $p = s = 1$

Example: Minimum Equilibrium



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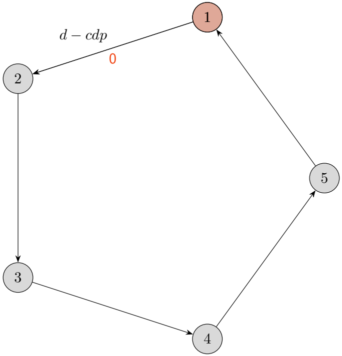

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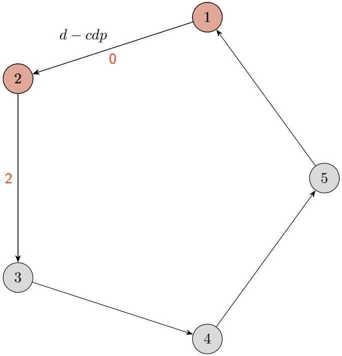

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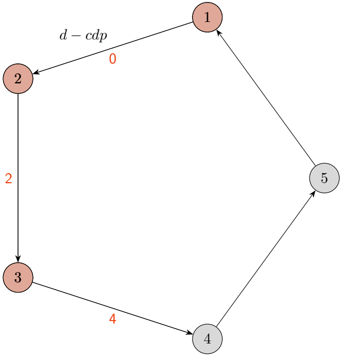

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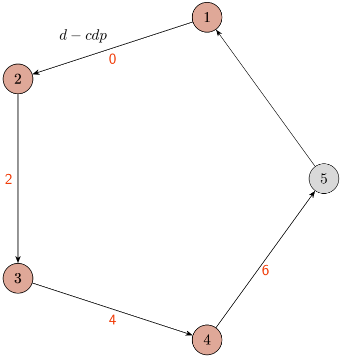

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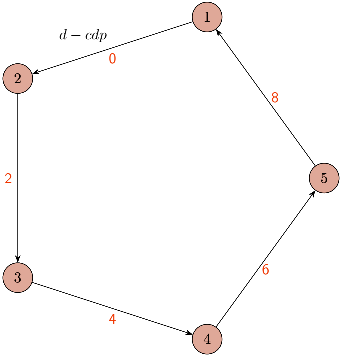

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- If $p = 0$, net debt amount is
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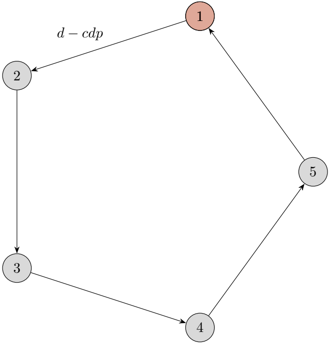
- All agents default and liquidate their long-term projects, and $p = 0$

Example: Intermediate Equilibrium



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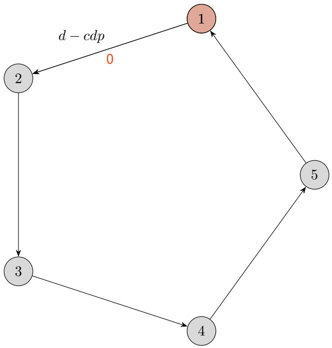

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- If $p = 0.25$, net debt amount is $d - cdp = 10 - 1 \times 10 \times 0.25 = 7.5$, and agent 2's remaining wealth is $2 + 2 \times 0.25 = 2.5$

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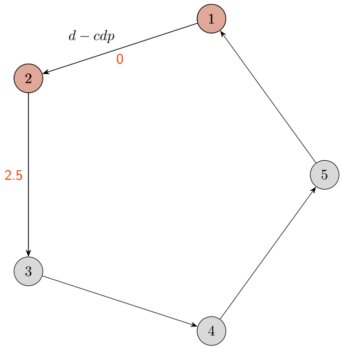

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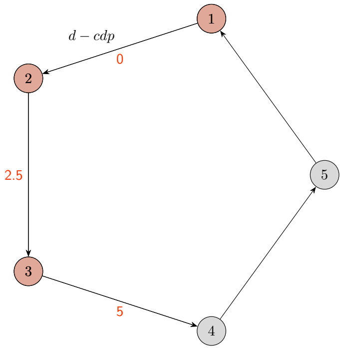

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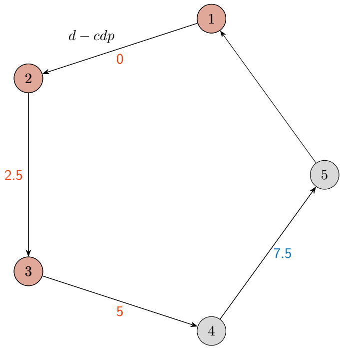
- If $p = 0.25$, net debt amount is $d - cdp = 10 - 1 \times 10 \times 0.25 = 7.5$, and agent 2's remaining wealth is $2 + 2 \times 0.25 = 2.5$

Example: Intermediate Equilibrium



- Each agent has
  $e_0 = 2$: 2 units of cash
  $h_0 = 2$: 2 units of assets
  $d = 10$: debt amount
  $c = 1$: collateral ratio
- Asset's fair value $s = 1$
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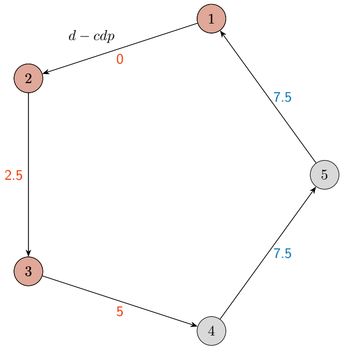

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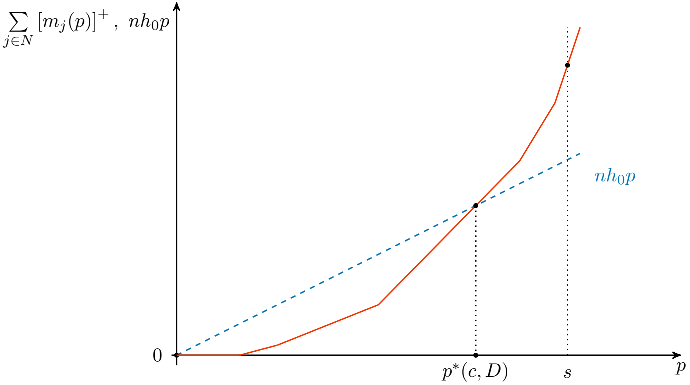
- Agent 5 has 2 cash to buy 8 assets at $p = 2/8 = 0.25$

Illustration of Multiple Equilibria



$$p = \min\left\{ \frac{\sum_{j \in N} [m_j(p)]^+}{nh_0}, s \right\}$$

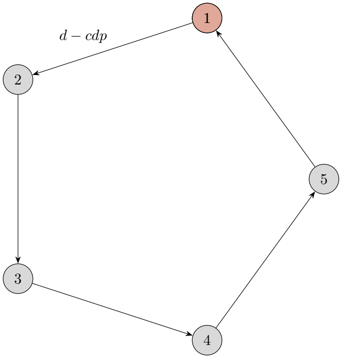
Main Result 2: Collateral Reuse Decreases Intermediate Equilibrium Price

Proposition

The intermediate equilibrium price $p^*(c, D)$ is decreasing in the degree of collateral reuse c regardless of the network structure D.

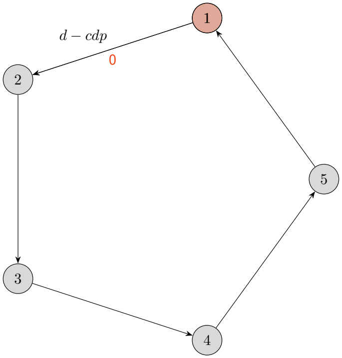
- $cdp \uparrow$ as $c \uparrow$, so $p \downarrow$ to equate the market clearing condition
- Aggregate net wealth becomes more sensitive to collateral price p
- Overall effect can lead to more defaults, as the aggregate net wealth in the intermediate equilibrium decreases

Intermediate Equilibrium Example: $c \downarrow \Longrightarrow p \uparrow$



- Each agent has
  $e_0 = 2$: 2 units of cash
  $h_0 = 2$: 2 units of assets
  $d = 10$: debt amount
  $c = 0.6$: collateral ratio
- Asset's fair value $s = 1$
- Agent 1 hit by liquidity shock

Intermediate Equilibrium Example: $c \downarrow \Longrightarrow p \uparrow$



- Each agent has
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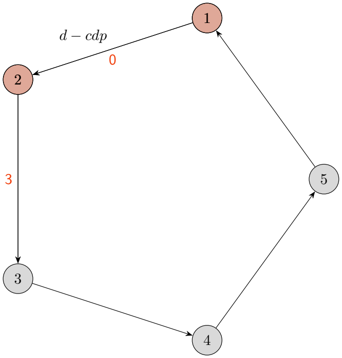
- Asset's fair value $s = 1$

- Agent 1 hit by liquidity shock

- If $p = 0.5$, net debt amount is $d - cdp = 10 - 0.6 \times 10 \times 0.5 = 7$, and agent 2's remaining wealth is $2 + 2 \times 0.5 = 3$

Intermediate Equilibrium Example: $c \downarrow \Longrightarrow p \uparrow$



- Each agent has
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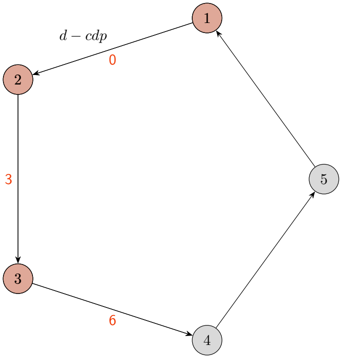
Intermediate Equilibrium Example: $c \downarrow \Longrightarrow p \uparrow$



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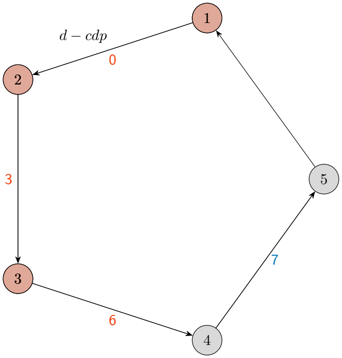
Intermediate Equilibrium Example: $c \downarrow \Longrightarrow p \uparrow$



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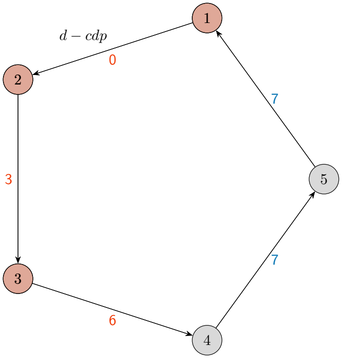

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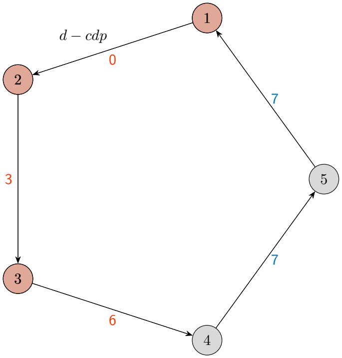
- If $p = 0.5$, net debt amount is $d - cdp = 10 - 0.6 \times 10 \times 0.5 = 7$, and agent 2's remaining wealth is $2 + 2 \times 0.5 = 3$
- Agent 4 and 5 have 1 and 2 cash, respectively, to buy 6 assets at $p = 3/6 = 0.5$

Intermediate Equilibrium Example: $c \downarrow \Longrightarrow p \uparrow$



- lower c $(1 \to 0.6)$
  but higher p $(0.25 \to 0.5)$

- Each agent has
  $e_0 = 2$: 2 units of cash
  $h_0 = 2$: 2 units of assets
  $d = 10$: debt amount
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- Asset's fair value $s = 1$

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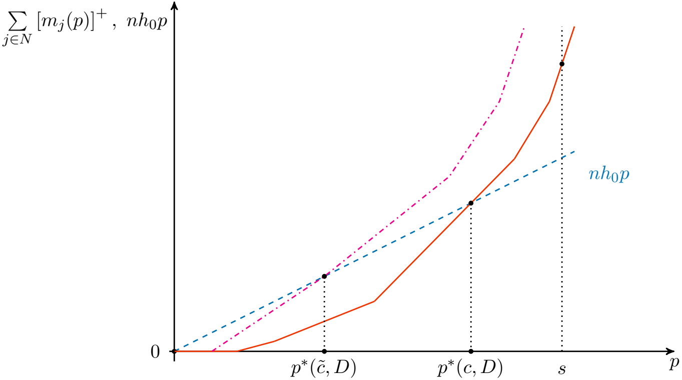
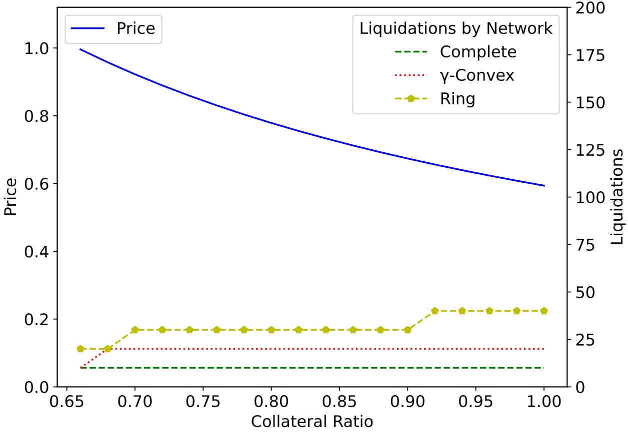
Illustration of Collateral Reuse and Changes in Equilibria



$$p = \min \left\{ \frac{\sum_{j \in N} [m_j(p)]^+}{nh_0}, s \right\}$$

Numerical Examples of Intermediate Equilibrium



- Even though the amount of liquidations can differ, the intermediate equilibrium price is the same across three networks

Equilibrium Selection

- Extend our model to incorporate equilibrium selection: which equilibrium out of the three will be realized?
- We consider both global games and best response dynamics
  - Results hold for both models of equilibrium selection
- Intermediate eqm price plays an important role in eqm selection

Equilibrium Selection and Global Games

- Building on top of the Global Games literature: Carlsson and Van Damme (1993), Morris and Shin (1998), Bernardo and Welch (2004), Goldstein and Pauzner (2005), Infante and Vardoulakis (2021), **Kuong (2021)**, Kashyap et al. (2024), Eisenbach and Phelan (2025)

- Strategic substitutability due to market forces (one agent's sales can be another agent's buying opportunity)

- Strategic complementarity arises due to payment obligations and contagion (cf. sequential servicing constraint, moral hazard and self-fulfilling fire sales)

Global Games Model

- Value of the asset payoff $\theta \sim U[0, \bar{\theta}]$, where $\bar{\theta} \geq s$

- Each agent receives a noisy signal $\theta_i = \theta + \psi_i$, $\psi_i \sim_{iid} U[-\psi, \psi]$

- Assume each individual agent is distributed on a continuum (a continuum of identical $i \in N$ with a mass of one)

Details

Main Result 3: Equilibrium Selection GG

Proposition

In the global games setup with $\psi \to 0$, the following hold:

1. *For any $\theta \geq p^*(c, D)$, the equilibrium price is $p = \theta$.*
2. *For any $\theta < p^*(c, D)$, the equilibrium price is $p = 0$, and all agents default.*
3. *The likelihood of the minimum equilibrium with $p = 0$ decreases as the degree of collateral reuse c increases.*

- The unique intermediate equilibrium price $p^*(c, D)$ is the threshold for equilibrium selection
- Any price above the threshold can satisfy the market clearing condition with $\sum_{j \in N} [m_j(\theta)]^+ \geq nh_0\theta$, for any $\theta \geq p^*(c, D)$
- Any price below the threshold leads to the minimum equilibrium as $\sum_{j \in N} [m_j(\theta)]^+ < nh_0\theta$ for any $0 < \theta < p^*(c, D)$

Best Response Dynamics

- Alternative equilibrium selection model based on Gilboa and Matsui (1991), Matsui (1992), and Mäder (2024)
- BRD: Each agent responds with a best response to all other agents' previous actions in each round
  $\Longrightarrow$ a best response path can converge to a Nash equilibrium
- Applicability: After observing a sudden price decline, agents who initially decided not to participate in the market, can and optimally choose to participate in the market
- Microfoundation of Walrasian tatônnement Details

Main Result 3': Equilibrium Selection BRD

Proposition

In the BRD-RB setup, the likelihood of realization of the minimum equilibrium is proportional to $p^*(c, D)$. Moreover, the likelihood of the minimum equilibrium decreases as the degree of collateral reuse c increases.

- **Basin of attraction** of an eqm is the set of all initial conditions giving rise to at least one BRD path ending in that eqm
- Basin of attraction of the maximum equilibrium is $(p^*(c, d), s]$
- As the degree of collateral reuse increases, the financial system is less likely to be perturbed by random noise and end up in $p = 0$

Illustration of Equilibrium Selection

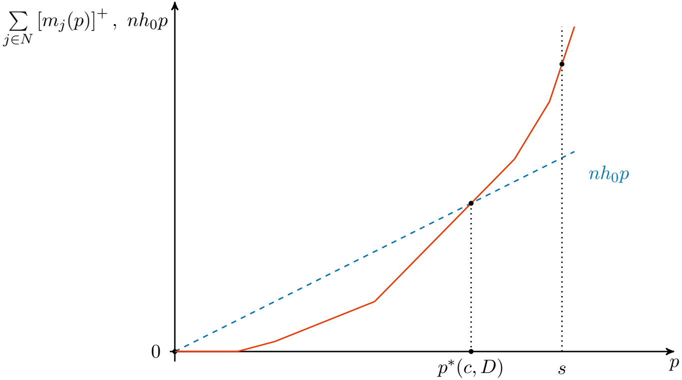
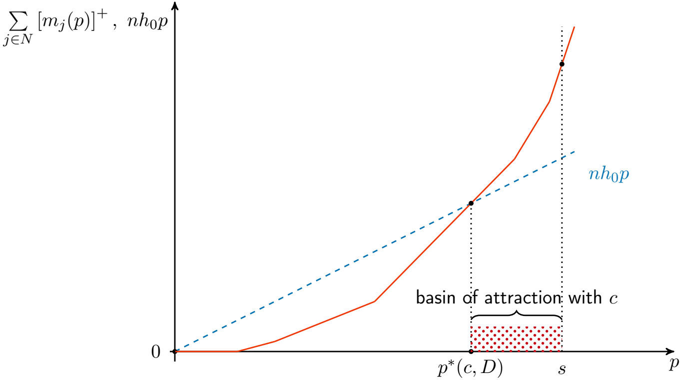
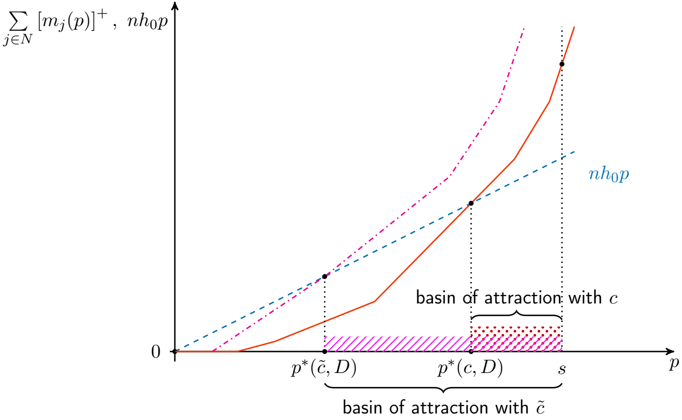


Illustration of Equilibrium Selection



- Any starting price within the basin of attraction will end up at $p = s$

Illustration of Equilibrium Selection



- Any starting price within the basin of attraction will end up at $p = s$

Collateral Reuse and Other Factors

- Degree of collateral reuse is often correlated with other factors: leverage, collateral circulation and scarcity, collateral runs and lender default, and length of lending chain (Gottardi et al., 2019; Infante, 2019; Park and Kahn, 2019; Infante and Vardoulakis, 2021; Chang, 2021; Maurin, 2022; Brumm et al., 2023; Infante and Saravay, 2024)

- We extend our model further to incorporate agents' endogenous risk taking behavior at $t = 0$

- All other factors can amplify our mechanism

- For simplicity, use BRD-RB equilibrium selection model (with uniformly distributed RB)

Risk-Taking Tradeoff

- Each agent has a cash endowment e_{-1} at the beginning of $t = 0$
- Agents allocate cash into
  - cash holdings e_0
  - investment in the long-term project ξ

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- Long-term project investment
  - Each unit of cash invested gives $R > 1$ at $t = 2$
  - If liquidated, the agent suffers *liquidation cost* $K(\xi)$
    $K(\xi)$ is convexly increasing ($dK/d\xi > 0$ and $d^2K/d\xi^2 > 0$)

Risk-Taking Tradeoff

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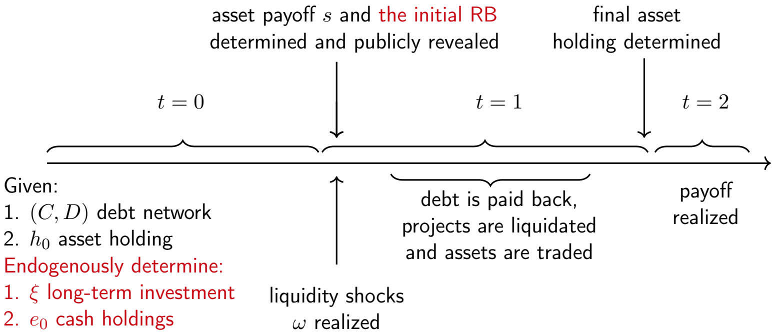
  - If liquidated, the agent suffers *liquidation cost* $K(\xi)$
    $K(\xi)$ is convexly increasing ($dK/d\xi > 0$ and $d^2K/d\xi^2 > 0$)
- Expected cost of liquidation:

$$p^*(c, D)K(\xi) + (s - p^*(c, D))\frac{1}{n}K(\xi)$$

Timeline with Endogenous Risk Taking



Agent's Optimization Problem at Date 0

$$\max_{e_0,\xi} -p^*(c,D)K(\xi) + (s - p^*(c,D))\left[\frac{n-1}{n}(\xi + \tilde{\pi}_j(e_0,D)) - \frac{1}{n}K(\xi)\right]$$

$$s.t. \qquad e_0 + \xi/R \leq e_{-1}$$

where $\tilde{\pi}_j(e_0,D) \equiv e_0 + h_0 s - (d - cds) + E_j\left[\sum_{j\neq i} x_{ji}(s)|\omega_j = 0\right]$ is the expected profit of agent j excluding the long-term investment payoff, when j is not shocked

- FOC:

$$\underbrace{\left[p^*(c,D) + (s - p^*(c,D))\frac{1}{n}\right]}_{\substack{\text{increasing in } p^*(c,D),\\ \text{hence decreasing in } c}} K'(\xi) = \underbrace{(s - p^*(c,D))}_{\substack{\text{decreasing in } p^*(c,D),\\ \text{hence increasing in } c}} \frac{n-1}{n}\left(1 - \frac{1}{R}\right)$$

- As c increases, agent j is less worried about liquidation cost

- Optimal investment amount ξ^* is increasing in c

Main Result 4: Fewer But More Severe Crises

Corollary (Empirical Prediction)

Holding all else equal, a higher degree of collateral reuse leads to a lower likelihood of crisis but a greater severity of crisis (lower social surplus) when it occurs.

- This result aligns well with the recent literature on excessive risk-taking behavior by individuals and systemic risk (Castiglionesi et al., 2019; Elliott et al., 2021; Galeotti and Ghiglino, 2021; Altinoglu and Stiglitz, 2023; Jackson and Pernoud, 2024; Shu, 2024)
- Our results demonstrate how endogenous risk-taking behavior can be exacerbated by an increase in collateral reuse

Results are Robust to Various Extensions

- Non-trivial liquidations ($\zeta > 0$)
- Multiple agents hit by liquidity shocks
- Size heterogeneity (scaling each agent's balance sheet differently)
- Non-regular network structures (e.g., star network)

Conclusion

- A model with both debt and collateral market contagion and endogenous fire-sale prices with multiple equilibria
- Even with the same degree of collateral reuse, there can be extreme swings in social surplus across equilibria
- Collateral reuse alone is not a concern for financial stability (lower likelihood of crises)
- Degree of collateral reuse can still negatively impact financial stability through its effects on risk-taking choices (lower likelihood but greater severity of crises)
- Supporting price and liquidity of collateral assets can be a critical policy intervention

Agent's Optimization Problem at $t = 1$

- Agent j maximizes long-term profit π_j at $t = 2$
- Decides: cash (e) and assets (h)
- Decisions subject to wealth, liabilities, and belief θ_j, which is s for now

$$\max_{e,h} \pi_j = e + hs + \xi \mathbb{1}\left\{a_j(p) > b_j(p)\right\}$$

$$\text{s.t.} \qquad e + hp = \left[a_j(p) - b_j(p)\right]^+$$

$$e \geq 0, \; h \geq 0$$

- Agent j will buy assets using all the available budget if $p < s$

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Harmonic Distance 1/2

Definition

The harmonic distance from agent i to agent j is

$$\mu_{ij} = 1 + \sum_{k \neq j} \left(\frac{d_{ik}}{d} \right) \mu_{kj},$$

with the convention that $\mu_{ii} = 0$ for all i.

- We focus on the set of networks such that any network has harmonic distances smaller than $\mu^*(0)$ when $p = 0$

Harmonic Distance 2/2

Proposition (Chang and Chuan (2024))

Suppose that agent j is under a negative liquidity shock of $\epsilon > ne_0$. Then, there exists $\mu^*(p) = (d - cdp)/(e_0 + h_0 p)$, and the following holds:

1. *If there is a nonempty set $\mathcal{S}$ such that agent $i \in \mathcal{S}$ does not default, then the equilibrium price is either $p = s$ or determined by*

$$\mathbb{1}' G \mu_{sj} = \frac{d - cdp}{e_0 + h_0 p} \mathbb{1}' G \mathbb{1} + \frac{n h_0 p}{e_0 + h_0 p},$$

   where μ_{sj} is the vector of harmonic distances from agents in $\mathcal{S}$ to j, G is a $|\mathcal{S}| \times |\mathcal{S}|$ non-singular M-matrix, and $\mathbb{1}$ is a vector of ones. Furthermore, if $\mu_{ij} < \mu^*(p)$, then agent i defaults.
2. *If all agents default, then the equilibrium price is $p = 0$ and $\mu_{ij} < \mu^*(0)$ for all i.*
3. *If $\mu^*(p) < 1$ for the equilibrium price p, then no other agents default.*

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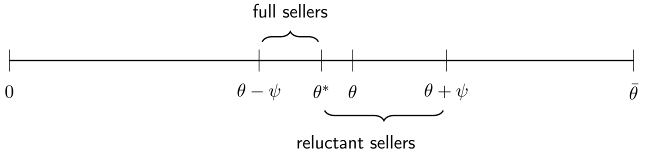
Global Games Supply and Demand



- Marginal buyer $\theta^* \in [\theta - \psi, \theta + \psi]$
- Full sellers: $\dfrac{\theta^* - (\theta - \psi)}{2\psi}$ **vs.** Reluctant sellers: $\dfrac{\theta + \psi - \theta^*}{2\psi}$
- Endogenous fire-sale amount of j in the entire continuum:

$$\phi_j(p) = \min\left\{ \frac{\theta + \psi - \theta^*}{2\psi} [h_j p - m_j(p)]^+ + \frac{\theta^* - (\theta - \psi)}{2\psi} h_j p, h_j p \right\}$$

- Endogenous demand:

$$\sum_{j \in N} \frac{\theta + \psi - \theta^*}{2\psi} [m_j(p) - h_j p]^+$$

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Intermediate Game of BRD

- Agent i in N maximizes its profits π_i by choosing an action $\alpha_i \in A = [0, h_0 p]$

- Strategic decision changes the fire sales amount:

$$\phi_i(p) = \min\left\{[h_0 p - m_i(p)]^+ + \alpha_i, h_0 p\right\}$$

- Agents continuously revise their actions for each $\tau \in [0, 1]$ (internal timing for the BRD path)

- A BRD path is a continuous and right-differentiable function $\alpha : [0, 1] \mapsto \mathcal{A}$ such that $\alpha'(\tau) \in \delta\left[\beta(\alpha(\tau)) - \alpha(\tau)\right]$ for some $\delta > 0$, for each $\tau \in [0, 1)$, and for any $\alpha(0) \in \Pi_{i \in N} A$

- Agents start with random beliefs (RB) $\kappa : \Theta \mapsto \Pi_{\in N} \Pi_{j \neq i} A$

Back