Collateralized Debt Networks with Lender Default*

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Abstract

Lender default arises with reuse of collateral, which is common in collateralized debt markets, because bankrupt lenders can default on their obligation to return the collateral and borrowers can suffer additional losses. I develop a network model with both borrower and lender defaults to analyze systemic risk in collateralized debt markets. The model endogenizes asset prices, leverage, and network formation simultaneously. The main mechanism of network formation is the borrowers' tradeoff between counterparty risk and leverage. Thus, a policy that eliminates the counterparty risk concern may have a hidden side effect to systemic risk. This side effect is novel because it does not exist if one of asset price, leverage, or network were exogenously given.

JEL classification: D52, D53, E44, G23, G24, G28

Keywords: collateral, reuse of collateral, default, financial networks

1. Introduction

Reuse (re-hypothecation) of collateral by the lender of a contract is very common in the market of collateralized debt, such as repurchase agreement (repo) contracts and derivatives (Fuhrer

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et al., 2016; Infante et al., 2018; Jank et al., 2021). The collateralized debt market is crucial to the whole financial system as the 2007-2009 financial crisis demonstrated (Gorton and Metrick, 2012; Copeland et al., 2014; Martin et al., 2014), and policy makers raised financial stability concerns related to the reuse of collateral in the market (Financial Stability Board, 2017). Therefore, understanding the underlying mechanism of the reuse of collateral is important in monitoring and mitigating the potential systemic risk stemming from the collateralized debt market.

The lender default problem also arises with reuse of collateral (Infante and Vardoulakis, 2021). A lender may be unable to return the reused collateral to the original borrower if the lender is under bankruptcy. For example, all of Lehman Brothers' assets, including borrowers' collateral, were frozen under the bankruptcy procedure in 2008. Many borrowers had to overcollateralize their positions to protect the lender (Lehman) in case of borrower default (Scott, 2014). While overcollateralization secured the lender's position, it exposed the borrowers to losses when they could not recover their collateral. The borrowers did not know when their collateral would be returned to them, nor did they know how much they would recover from the bankruptcy process, and they paid a sizable cost throughout the recovery (Fleming and Sarkar, 2014). Another example is MF Global, a broker-dealer that went bankrupt in 2011. The bankruptcy procedure took five years to resolve all the borrowers' claims. The borrowers had to go through the lengthy process with considerable costs to stay involved and could not access their collateral assets (SIPC, 2016).

Based on these motivating facts, the research questions of this paper are the following: How do participants of the collateralized debt market borrow from each other when they can reuse collateral and default as lenders? How is the underlying systemic risk determined? How do the structure of the market and systemic risk change under different market conditions, including a change in regulation?

The main contribution of this paper is the development of a new model that sheds light on these questions. I develop a model that combines the frameworks of financial network and general equilibrium to fill in the gap in the literature. A debt network keeps track of how agents borrow from each other as well as how the collateral flows through the reuse of collateral. The general equilibrium forces determine asset prices and relevant returns of each investment option for agents. Therefore, the model determines asset prices, leverage, and network formation all endogenously. This paper is the first attempt to endogenize all three of them simultaneously. The main mechanism and policy implication of this paper do not exist if one of them were exogenous.

This combination of financial network and general equilibrium is important in analyzing the contagion and amplification of a shock in the collateralized debt market. A typical collateralized debt contract takes the form of a one-to-one interaction between two counterparties—a borrower and a lender. Thus, a collateralized debt network, the collection of such one-to-one relationships, has two transmission channels of shocks—the price (of the collateral) channel and the counterparty channel. Therefore, the interaction of the two channels of contagion is essential in understanding the systemic risk in the collateralized debt market.

For example, the collapse in the prices of subprime mortgages in 2008 had a direct effect on many financial institutions that held related assets either from outright purchase or as collateral, but the resulting bankruptcy of the Lehman Brothers, which spread the losses to Lehman's counterparties, exacerbated the initial shock (Singh, 2017). These counterparty losses triggered fire sales of assets, which made prices decline even further (Demange, 2016; Duarte and Eisenbach, 2021; Duarte and Jones, 2017). Even U.S. Treasury securities, one of the safest assets and most commonly used collateral in the world, can experience a price decline when there is a crash such as the COVID-19 pandemic.[1] Therefore, a model that incorporates the interaction between price and counterparty channels is necessary to capture the full picture of the crisis in collateralized debt markets (Glasserman and Young, 2016).

The model is as follows. There are n agents who trade an asset that can be used as collateral in a competitive market. Agents trade because they disagree on the fair value of the asset ex ante. Agents enter into bilateral contracts specifying the amounts of debt and collateral. The lender of a debt contract can *reuse* the collateral to borrow money from someone else. A network of the amounts of debt and collateral represents all the collateralized debt contracts. Agents are subject to *liquidity shocks* before paying back their debt. Because of liquidity shocks, agents may go bankrupt. Both borrower and lender defaults are incorporated in the model. In particular, when the lender fails to return the collateral, the borrower has to go through a costly process to recover the collateral from the lender. This lender default generates additional propagation through a *counterparty channel*, whereas price changes in the asset market affect agents' net worth as a *price channel*.

The main insight of the model is that network formation is based on the tradeoff between counterparty risk and leverage. Borrowers would prefer to maximize their (contract-level) leverage (or minimize margin) to maximize their return. If there is no lender default loss, then agents who purchase the asset would borrow from the most favorable lender to them. Then, the lenders of the contracts would reuse the collateral to borrow from their own most favorable lenders, and the lenders' lenders would do the same, and so on. Therefore, a single-chain network—that is, agent j borrows exclusively from agent $j+1$ for all $j < n-1$ as in figure 1— is formed endogenously. However, if there are lender default risks, borrowers would diversify their lenders and form a multi-chain network as in figure 2. The tradeoff between counterparty risk and leverage exists because borrowers have to deal with more restrictive lenders, who lend less for the same collateral. Therefore, an increase in counterparty risk leads to an increase in the number of counterparties and a decrease in leverage and reuse of collateral, because the borrowers borrow more directly from the ultimate lenders rather than indirectly through intermediaries.

The model also shows that there is an amplifying interaction between the two channels of contagion. Bankruptcy of an agent affects its counterparties through the counterparty channel. Asset prices also go down because of these counterparty losses. Then, all agents in the market experience mark-to-market losses in their balance sheet through the price channel. This price decline can lead to more bankruptcy causing an increase of the counterparty channel of contagion

[1] In the inter-dealer repo markets, even U.S. Treasuries have positive haircuts (Duarte and Eisenbach, 2021).

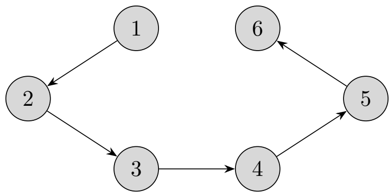

Figure 1: Single-chain network

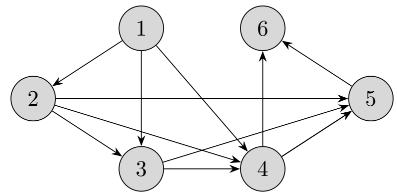

Figure 2: Multi-chain network

that feeds back into the price channel amplification again and so on. Therefore, a change in an agent's behavior can have an amplified effect on all agents in the market.

There are positive externalities from diversification of counterparties. Diversification reduces not only individual counterparty risk, but also systemic risk by limiting the propagation of shocks and price volatility due to lower leverage. If an intermediary becomes safer, then its borrowers become safer as well. In addition, a lower level of debt leads to lower price volatility, making each agent's balance sheet more stable.

The main policy implication of this paper is that a policy change mitigating counterparty losses can exacerbate the diversification externality problem. For example, mandating all trades to be cleared by a central counterparty (CCP) could have an unexpected side effect. The policy change will distort the tradeoff between counterparty risk and leverage by eliminating or mitigating the counterparty risk concerns. Therefore, agents will form a single-chain network, which maximizes leverage, reuse of collateral, and systemic risk. Although a policy change can bring many positive effects that reduce systemic risk, a countervailing force can be caused by the endogenous network responses. Therefore, this result highlights the importance of regulating counterparty exposure concentration such as the single counterparty credit limit rule of the Dodd-Frank Wall Street Reform and Consumer Protection Act.

These results would not exist if one of leverage, asset prices, or network were exogenous, because then there would be no tradeoff between counterparty risk and leverage. Also, a simple three-agent model cannot capture the effect of diversification of a lender properly, as there are no lenders to diversify across. Hence, this paper finds a novel feature of endogenous response in the network structure related to systemic risk.

Finally, the predictions of the model align well with the empirical observations in the literature, filling in the gap between the theoretical literature and the recent empirical literature. First, the existing models focusing only on borrower default predict a strong negative relationship between haircuts and interest rates, because a lower haircut implies higher borrower default risk. In contrast, the data do not find a strong significant relationship between the two (Baklanova et al., 2019). In my model, the relationship can be weak because of high lender default risk, which lowers the incentives of the borrower to borrow at a high haircut or high rate. Second, the standard models predict a single haircut used in the market for the same asset. However, the data show that multiple haircuts are used for the same CUSIP (Committee on Uniform Security Identification Procedures) level asset (Baklanova et al., 2019). In my model, there can be multiple haircuts for the same asset because they are traded across different counterparties multiple times. Third, the existing models have preset roles in reuse of collateral and do not predict how the portfolio of counterparties is determined. The pattern of reuse changes with the underlying market conditions in empirical observations (Financial Stability Board, 2017; Singh, 2017; Infante and Saravay, 2020; Jank et al., 2021). Furthermore, the debt network changed significantly after the Lehman bankruptcy over both the short and long term (Craig and Von Peter, 2014; Eren, 2015; Sinclair, 2020). The tradeoff between the counterparty risk and leverage in my model naturally predicts such changes under different market conditions. Moreover, high levels of reuse would lead to a higher volatility of rates in my model as observed in the data (Jank et al., 2021).

1.1. Relation to the Literature

The first contribution of this paper is developing a model that incorporates both the counterparty and price channels of contagion with an endogenous network formation, which is the first attempt in the literature. No major institution failed because of losses on its direct exposures to Lehman; thus, developing a model that combines different shock transmission channels in financial networks is important (Upper, 2011; Glasserman and Young, 2016). The interaction of the two channels leads to a novel network formation mechanism that is new to the literature.

The counterparty contagion through financial networks in this paper is based on the insights from the literature following Eisenberg and Noe (2001) and Acemoglu et al. (2015). This paper also incorporates discontinuous jumps in the payoffs in case of bankruptcy as in Elliott et al. (2014). The endogenous network formation is based on portfolio decisions similar to Allen et al. (2012). The insight of externalities coming from counterparty risk exposure is similar to Zawadowski (2013). This paper contributes to the literature by incorporating externalities from network formation.

The endogenous price determination in this paper is based on the literature on general equilibrium with collateralized debt. The literature following Geanakoplos (1997) was developed in Geanakoplos (2003), Geanakoplos (2010), Simsek (2013), and Fostel and Geanakoplos (2015), which introduce models with collateral, explore how heterogeneity can generate collateralized debt and trade, and show how endogenous (contract-level) leverage is determined. In particular, Gottardi and Kubler (2015) and Geerolf (2018) introduce pyramiding—that is, using a contract backed by collateral as collateral—which is similar to the reuse of collateral in this paper. This paper contributes to this literature by linking these features into the network formation mechanism and analyzing the effect of counterparty risks instead of treating all trades as fully diversified anonymous trades.

In particular, cash holdings and endogenous asset prices in this paper counteract the incentives to correlate payoffs. Many financial network models have an equilibrium in which agents have overlapping asset and counterparty portfolios or a common correlation structure (Allen et al., 2012; Cabrales et al., 2017; Elliott et al., 2021; Erol and Vohra, 2020; Jackson and Pernoud, 2021). In such models, agents have strong incentives to correlate their payoffs with those of their counterparties, because they can enjoy better payments from their counterparties when they are solvent while being insolvent when they expect lower payments from others. However, this paper introduces an opposing force to such incentives, which is the marginal utility of cash coming from general equilibrium forces. Agents do not hold correlated portfolios because, if everyone else collapses, then the one who survives can make a huge return by purchasing all the remaining cheap assets.

The feedback from agents' wealth to collateral price is crucial in this paper. Other papers consider the interaction between counterparty and price channels, such as Capponi and Larsson (2015), Cifuentes et al. (2005), Di Maggio and Tahbaz-Salehi (2015), Gai et al. (2011), Ghamami et al. (2021), and Rochet and Tirole (1996). This paper differs by incorporating an endogenous network formation with the price channel for the underlying collateral.

Allen and Gale (2000), Babus (2016), Babus and Hu (2017), Babus and Kondor (2018), Brusco and Castiglionesi (2007), Capponi and Larsson (2015), Chang and Zhang (2021), Elliott et al. (2021), Erol and Vohra (2020), Farboodi (2017), and Freixas et al. (2000) studied endogenous network formations in financial networks. They consider the endogenous network structure and possible inefficiencies and systemic risks. Unlike the models in these papers, the network formation in this paper is based on general equilibrium forces rather than game-theory-based forces such as pairwise stability.

This paper is also related to the literature on lender default that shows how collateral can act as a contagion channel to the borrower. Gottardi et al. (2019), Infante and Vardoulakis (2021), Infante (2019), and Park and Kahn (2019) investigated the lender default problem in collateralized lending and relevant deadweight loss. This paper incorporates the lender default feature into the endogenous network structure. Also, the same collateral can be reused for an arbitrary number of times in contrast to other models of reuse of collateral.

1.2. Motivating Example

Figure 3 is an example of the flow of cash and collateral for a collateralized debt contract. The left figure visualizes the transaction at $t = 0$, where borrower b posts c amount of collateral to the lender l and l lends cash in the amount of $cq(d)$ to b. If the price of the asset p_1 is greater than the promise d at $t = 1$, then b pays the promise and l returns the collateral, as seen in the top-right figure. If the price p_1 is lower than d, then the borrower defaults and l keeps the collateral, as in the middle-right figure, assuming nonrecourse debt for simplicity. If the lender l is bankrupt, then b suffers cash loss in retrieving collateral even when b pays the promised amount of cd, as in the bottom-right figure. This lender default loss is increasing in b's collateral exposure to l.

Reuse of collateral is prevalent in a wide variety of collateralizable assets (Singh, 2017; Infante and Vardoulakis, 2021). In reality, borrowers prefer to allow reuse of their collateral. Even after the fall of the Lehman Brothers, most borrowers continued to allow reuse of their collateral (Singh, 2017) because reuse of collateral generates more liquidity for the borrowers themselves.

Figure 4 exemplifies the effect of reuse of collateral. Now suppose that there is an intermediary i, all three agents have the same cash endowment of 50, and they have different beliefs over the fundamental value of the asset as $s_l = 40$, $s_i = 80$, and $s_b = 100$, respectively. Also, suppose

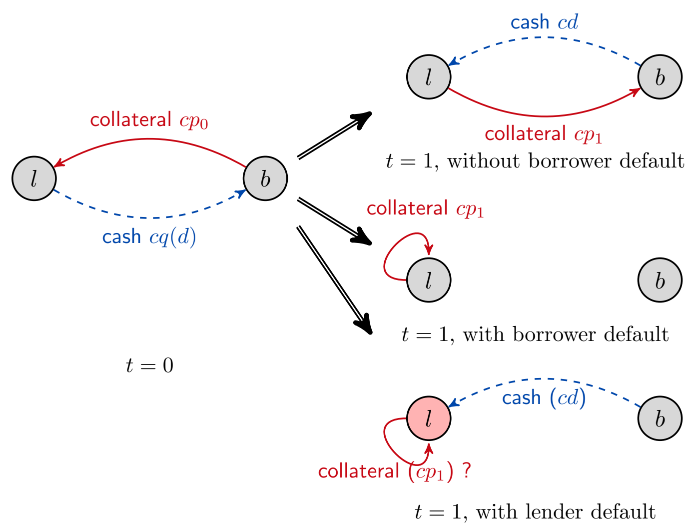

Figure 3: Flows of cash and collateral for three cases

Note: The blue dashed arrows represent flows of cash, and the red arrows represent flows of collateral. The left figure shows the flows in $t = 0$. The top-right figure shows the flows in the case without borrower default in $t = 1$, the middle-right figure shows the flows in the case with borrower default in $t = 1$, and the bottom-right figure shows the flows in the case with lender default in $t = 1$.

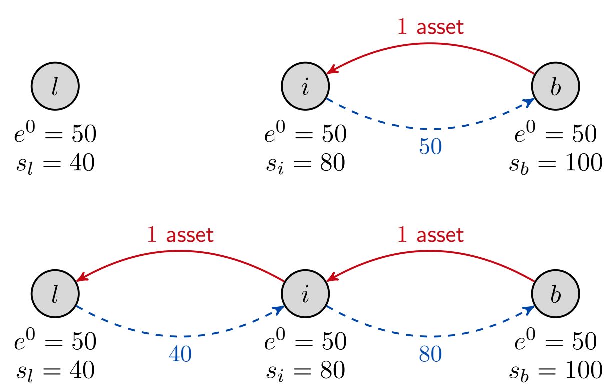

Figure 4: Example of effect of reuse

Note: The blue dashed arrows represent flows of cash, and the red arrows represent flows of collateral. The top figure represents the case of borrowing 50 from i, and the bottom figure represents the case of borrowing 80 from i, who reuses the collateral and borrows 40 from l again.

there is no risk in $t = 1$ and the interest rate is zero. Borrower b is the most optimistic agent and would like to buy as much of the asset as possible. Agent b can increase the amount of asset purchase by leveraging more. When borrowing from lender l, agent b would not offer a promise above 40, because agent l believes the asset is worth 40, so any promise greater than 40 will still deliver 40 because of borrower default. Therefore, the maximum amount of cash that b can borrow is 40. If agent b borrows from agent i, then b will promise up to 80, which provides b a higher leverage than the leverage of borrowing from l. However, because agent i's endowment of cash is only 50, the maximum amount that i lends to b is 50 without reuse of collateral. In contrast, if i can reuse the collateral, then i can borrow 40 from l. Now the effective cash available for i becomes $50 + 40 = 90$, and b can borrow 80 from i, which is greater than the borrowing amount of 50 under no reuse. The leverage of b with no reuse is $100/(100 - 50) = 2$, while the leverage of b with reuse is $100/(100 - 80) = 5$.[2] Therefore, agent b can increase leverage by 150 percent by reuse.

The main results of this paper require more than three agents in the model because diversification of lenders for the intermediate agents does not exist in a three-agent model case such as in figure 4, as agent i does not have any other counterparty than agent l from which to borrow.

Now extend the example and include another agent m, who is even more pessimistic about the asset as $s_m = 30$. Then, the lender l can also reuse the collateral and leverage the position by borrowing from the ultimate lender m. In the absence of lender default risk, the equilibrium debt network becomes the top panel of figure 5. However, if lender default risk becomes relevant, b may not want to concentrate all the collateral exposures to lender i only. Borrower b can diversify across i, l, m, while putting more weights on the counterparty that could provide higher leverage, as in the bottom panel of figure 5. Similarly, i may also diversify across l and m, reducing the counterparty risk. This move could also help b as i becomes safer, lowering the probability of lender default by i. Furthermore, diversification decreases the overall leverage of the market, lowering the volatility of the asset price. Therefore, the diversification externality depicted in figure 5 shows the importance of analyzing the general model suggested in this paper instead of a simple three-agent model.

2. Model

2.1. Goods and Agents

There are three periods: $t = 0, 1, 2$. There is a single consumption good, cash, that is storable and denoted as e. There is a divisible asset that generates a cash payoff at $t = 2$ and is denoted as a. The true asset payoff, $s \in [\underline{s}, \bar{s}] \subset \mathbb{R}^+$, is revealed to everyone at the beginning of $t = 1$. The price of the asset is denoted as p_t for each $t = 0, 1, 2$, and the price of cash is normalized to 1 for each period. Denote $\tilde{p}_t$ as the price as a random variable at t.

The set of agents is $N = \{1, 2, \ldots, n\}$. Agent j's subjective prior distribution of the asset payoff s is F_j, which may differ across agents.[3] This belief disagreement is to generate trades across agents. Each agent is endowed with the same e^0 amount of cash and zero amount of asset at $t = 0$. There are A amounts of assets held by external un-modeled agents who sell all of their assets and disappear at the end of $t = 0$.[4] The common utility function of agents is linear to their terminal cash holdings at $t = 2$; therefore, agents are risk neutral.

[2]The leverage here is calculated as (asset price)/(haircut), as the interest rate is zero.

[3]For the agent index, j is used throughout the paper to use i for the lender index and k for the borrower index.

[4]This assumption, also used in Simsek (2013), is only to shut down the feedback from the asset prices to the net worth of agents. All main results in this paper go through without this assumption.

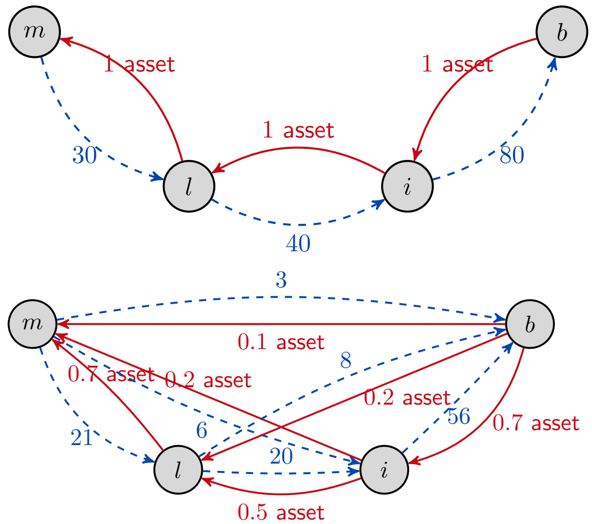

Figure 5: Example of tradeoff between counterparty risk and leverage

Note: The blue dashed arrows represent flows of cash, and the red arrows represent flows of collateral. The top figure represents the single-chain network, which arises when there is no lender default risk. The bottom figure represents the multi-chain network, which arises when there is lender default risk and borrowers diversify their lenders.

In the beginning of $t = 1$, each agent $j \in N$ can receive a negative liquidity shock with probability θ_j.[5] The size of the liquidity shock ϵ_j is independent and identically distributed across $j \in N$ with distribution function G, which is differentiable in support $[0, \bar{\epsilon}]$, and g is its density function. Assume that the upper bound of liquidity shock is large enough, $\bar{\epsilon} > e^0 + A\bar{s}$. Denote that $\epsilon_j = 0$ if j did not receive a liquidity shock at $t = 1$.[6]

2.2. Collateralized Debt Network and Markets

Agents can borrow or lend cash through a one-period (collateralized) debt contract using the asset as collateral at $t = 0$.[7] A borrowing contract comprises the amount of collateral posted c_{ij}, the debt amount per one unit of collateral d_{ij}, and the identities of the lender and the borrower i, j. All borrowing contracts are nonrecourse, so the borrowers can default on their promised debt amount with no consequences. Thus, the actual debt payment per unit of collateral from borrower j to lender i is $\min\{d_{ij}, p_t\}$, because borrowers will give up their collateral when the price of the collateral is less than the promised debt amount. Denote $q_i(d)$ as the price of the contract or the amount of cash lender i lends at $t = 0$ to a borrower who promises d per unit of collateral at $t = 1$. The gross interest rate is $d_{ij}/q_i(d_{ij})$, and the haircut is $(p_0 - q_i(d_{ij}))/p_0$.

The collateral posted by a borrower is held by the lender who can reuse it to borrow cash from

[5] This liquidity shock can be interpreted as senior debt or withdrawal of deposit as in Diamond and Dybvig (1983), or as a productivity shock as in Acemoglu et al. (2015) and Elliott et al. (2021).

[6] Note that $\epsilon_j = 0$ is a measure zero event if j received a shock.

[7] No contract will be traded in $t = 1$ because there is no additional uncertainty and endowment at $t = 2$.

someone else. Let a_j^1 denote the amount of asset agent j holds at $t = 1$ by purchasing at $t = 0$. Each agent j should satisfy the collateral constraint $a_j^1 + \sum_{k \in N} c_{jk} \geq \sum_{i \in N} c_{ij}$. The constraint implies that the collateral agent j is posting should be coming from either agent j's outright asset purchase or reuse of collateral that agent j's borrowers posted to j.

A (collateralized) *debt network* at $t = 0$ is a weighted directed multiplex (multilayer) graph formed by nodes N and links with two layers $\alpha = 1, 2$ defined as $\vec{\mathcal{G}} = \left(\mathcal{G}^{[1]}, \mathcal{G}^{[2]}\right)$, where $\mathcal{G}^{[\alpha]} = \left(N, L^{[\alpha]}\right)$, $L_{ij}^{[1]} = c_{ij}$, and $L_{ij}^{[2]} = d_{ij}$. Define the adjacency matrices $C = [c_{ij}]$ and $D = [d_{ij}]$ as collateral matrix and contract matrix, respectively. For a fixed N, a debt network can be represented by a double of (C, D) and describes how much each agent borrows from or lends to other agents. Following the convention, set $c_{ii} = d_{ii} = 0$.

If lender i has negative wealth (net worth) at $t = 1$, then i goes bankrupt and defaults on contracts. The borrower j can also default and forgo the collateral if j prefers to do so. However, if j wants to retrieve the collateral, j suffers a lender default loss in the *cash* amount of

$$\Psi_{ij}(C)[p - d_{ij}]^+, \tag{1}$$

where Ψ_{ij} is a function of the collateral matrix and $[p - d_{ij}]^+$ is the difference in value between the price of the collateral and the debt with $[\cdot]^+ \equiv \max\{\cdot, 0\}$. The function Ψ_{ij} is a reduced-form representation of the severity of the lender default. The Ψ_{ij} can represent the fraction of collateral lost, the litigation cost for the borrower that may depend on the collateral exposure, or the opportunity cost of time from the delay in delivery of the collateral, as discussed in Section 4.1.

The markets for goods and contracts are competitive Walrasian markets for $t = 0, 1, 2$. Agents agree to disagree, and the full structure of the belief disagreement, including each agent's subjective belief, is common knowledge as in Fostel and Geanakoplos (2015) and Simsek (2013). Therefore, agents are price-takers and know each other's belief and liquidity shock. [8]

2.3. Timeline

The timeline of the model, depicted in figure 6, is the following. Agents are endowed with cash at the beginning of $t = 0$. Agents buy assets from external agents and form a debt network (C, D) by borrowing from and lending to each other at $t = 0$. At the beginning of $t = 1$, liquidity shocks $\epsilon \equiv (\epsilon_1, \ldots, \epsilon_n)$ are realized and true asset payoff s is revealed. All agents update their beliefs accordingly to the true s. All the debt is paid back during $t = 1$, either by the promised amount or by giving up the collateral. An agent $j \in N$ may have ϵ_j that is greater than j's net worth—that is, net cash and asset holdings multiplied by the market price—so j goes bankrupt. The collateral is returned to the borrower from the lender, but some borrowers may suffer additional lender default losses. At the end of $t = 1$, agent j's final asset holdings a_j^2 are determined. At $t = 2$, payoffs of

[8] This assumption follows the tradition of the general equilibrium literature. One way to interpret this assumption is to consider that each agent j consists of a continuum (or hundreds) of homogeneous agents within the same type of j with perfectly correlated uncertainties—that is, all j agents receive the same liquidity shocks (otherwise there will be no agent-level uncertainty due to the law of large numbers).

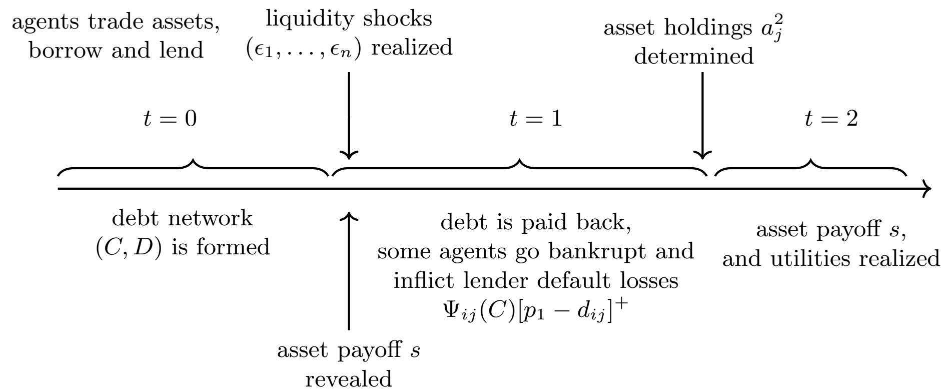

Figure 6: Timeline of the model

the asset are realized, and agents gain utility from cash.

3. Optimization Problem and Equilibrium Concept

Now that all the model structure is defined, the agents' optimization problem and equilibrium can be defined. I begin by defining the model backwards. Agents have no optimization problem at $t = 2$ because there are no additional actions and endowments. I define the agents' decision problem at $t = 1$ and the intermediate equilibrium concept of *payment equilibrium*. Then, I define the optimization problem of agents at $t = 0$ and the full equilibrium concept, *network equilibrium*, at $t = 0$.

3.1. Payment Equilibrium at Period 1

At $t = 1$, agents receive liquidity shocks ϵ and pay each other their debt and inflict lender default losses $\Psi \equiv [\Psi_{ij}]_{i,j \in N}$ for a given debt network (C, D), cash holdings $e^1 \equiv (e_1^1, e_2^1, \ldots, e_n^1)'$, asset holdings $a^1 \equiv (a_1^1, a_2^1, \ldots, a_n^1)'$, and revealed asset payoff s. Simultaneously, agents also trade in a Walrasian market, and the asset price p_1 is determined endogenously. Bankrupt agents have to sell all their assets, and only the surviving agents can buy the assets.

For any given ϵ and p_1, agent j's (nominal) wealth relevant to market clearing is

$$m_j(p_1) = e_j^1 - \epsilon_j + a_j^1 p_1 + \sum_{i \in N} (c_{ji} \min\{p_1, d_{ji}\} - c_{ij} \min\{p_1, d_{ij}\}) - \sum_{i: m_i < 0} \Psi_{ij}(C)[p_1 - d_{ij}]^+. \quad (2)$$

If $m_j(p_1) < 0$, j goes bankrupt, belongs to the bankruptcy set $B(\epsilon|s)$, and exits the market.

If $p_1 < s$, the return of the asset, s/p_1, exceeds the return of cash, which is 1. Thus, agents would spend all their cash to buy the asset, and the market price is determined by cash-in-the-market pricing. The asset holding a_j^2 is determined by $a_j^2 = [m_j(p_1)]^+/p_1$. If $p_1 = s$, a_j^2 becomes irrelevant due to equivalence of returns between cash and asset.

The cash value of the aggregate supply is Ap_1. The aggregate cash value of surviving agents in the market is $\sum_{j\in N}[m_j(p_1)]^+$. Therefore, the market clearing condition is

$$\sum_{i\in N}[m_i(p_1)]^+ = Ap_1 \quad \text{if } 0 \le p_1 < s, \tag{3}$$
$$\sum_{i\in N}[m_i(p_1)]^+ \ge Ap_1 \quad \text{if } p_1 = s. \tag{4}$$

Hence, an equilibrium is determined by the wealth vector $m \equiv (m_1, \dots, m_n)$ and the resulting market price p_1 of the asset. This market clearing price and allocation is defined as payment equilibrium, which is an intermediate equilibrium of $t = 1$ as follows.

Definition 1. *For a given period-1 economy of $(N, C, D, e^1, a^1, \epsilon, s, \Psi)$, a **payment equilibrium** is (m^*, p_1^*), where m^* is the wealth vector and p_1^* is the asset price, that simultaneously satisfies wealth equation (2) and market clearing conditions (3) and (4).*

3.2. Network Equilibrium at Period 0

Each agent maximizes their expected payoff in $t = 2$ at the beginning of $t = 0$ by choosing an investment portfolio. Each agent $j \in N$ can hold cash, in the amount of e_j^1; purchase the asset and carry it to the next period, in the amount of a_j^1; borrow from agent $i \in N$, posting collateral in the amount of c_{ij} and promise cash per collateral as d_{ij} while receiving $q_i(d_{ij})$ per collateral; or lend to agent $k \in N$, receiving collateral in the amount of c_{jk} for the promised cash per collateral as d_{jk} while paying $q_j(d_{jk})$ per collateral.

For a given portfolio, the agent's expected wealth in $t = 1$ is determined. However, wealth should be evaluated by the marginal utility of cash for each state, s/p_1 that could be greater than 1 if $p_1 < s$. Agent j's nominal wealth and marginal utility of cash depend on realization of liquidity shocks ϵ and asset payoff s. Agent j's maximization problem becomes

$$\begin{aligned} &\max_{\substack{e_j^1, \{c_{ij}, d_{ij}\}_{i\in N},\\ a_j^1, \{c_{jk}, d_{jk}\}_{k\in N}}} E_j\left[[m_j(p_1)]^+ \frac{s}{p_1}\right] \\ \text{s.t. } & a_j^1 + \sum_{k\in N} c_{jk} \ge \sum_{i\in N} c_{ij}, \\ & e^0 = e_j^1 - \sum_{i\in N} c_{ij} q_i(d_{ij}) + \sum_{k\in N} c_{jk} q_j(d_{jk}) + a_j^1 p_0, \end{aligned} \tag{5}$$

where the first constraint is the collateral constraint and the second constraint is the budget constraint.

The equilibrium concept used throughout the paper is a hybrid version of general equilibrium with price functions that are affected by the network structure as follows.

Definition 2 (Network Equilibrium)**.** *For a given economy $(N, (s_j, \theta_j, e^0)_{j\in N}, A, \Psi, G)$, a septuple $(C^*, D^*, e^{1*}, a^{1*}, p_0^*, \tilde{p}_1^*, q^*)$ where $C^*, D^* \in \mathbb{R}_+^{n\times n}$, $e^{1*}, a^{1*} \in \mathbb{R}_+^n$, $p_0^* \in \mathbb{R}_+$, functions $p_1^* : \mathbb{R}_+^{n+1} \to$*

$\mathbb{R}_+$ *and* $q^* \equiv (q_1^*, \ldots, q_n^*)$ *with* $q_j^* : \mathbb{R}_+ \to \mathbb{R}_+$ *is a* ***network equilibrium*** *if*

1. $(C^*, D^*, e^{1*}, a^{1*})$ *solves the agent maximization problem with two constraints,*
2. *markets are cleared as* c_{ij}^* *is optimal for both agent* i *and* j *for all* $i, j \in N$*,*
3. *asset market clears as* $\sum_{j \in N} a_j^1 = A$*,*
4. *asset price* $\tilde{p}_1$ *is determined by the payment equilibrium for each* (ϵ, s)*,*
5. *and asset price* p_0 *and contract prices* q *are determined by no-arbitrage conditions.*

3.3. Discussion of the Model

Borrower Default. Borrower default results in a costless transfer of collateral to the lender. For example, typical repo contracts are exempt from automatic stay of bankruptcy provisions or even involve ownership transfer to the lender at the start of the transaction. The model simplifies borrower default, as the debt contracts are nonrecourse. One reason for this assumption is to shut down the complexity of borrower default to focus on lender default. Another reason is to have a tractable model of network formation.[9] Although debt payments are independent from borrowers' balance sheets, the price channel still affects payments. This case is not distant from reality because the role of collateral is exactly to minimize the borrower default exposure. The no-recourse contract assumption is also commonly used in the literature as in Geanakoplos (2010), Simsek (2013), Fostel and Geanakoplos (2015), and Geerolf (2018) for similar reasons.

Lender Default. Lenders cannot strategically default and still fulfill their obligation to return collateral unless they are bankrupt, which is in line with the actual markets. However, a bankrupt lender can inflict additional losses to its counterparties. This lender default loss, similar to the counterparty default losses that are prevalent in the literature, can be in terms of time, effort, litigation cost, or congestion costs, which are deadweight loss to the economy. For example, over 100 hedge funds had prime brokerage accounts or debt obligations under Lehman Brothers, and these accounts were frozen during the bankruptcy of Lehman Brothers. These positions, valued at more than $400 billion, were frozen, which further exacerbated the liquidity shortage of the market (Lleo and Ziemba, 2014).[10]

Expectations. Each agent j's expectation is based on the subjective belief on asset payoff s_j and the distribution of liquidity shocks ϵ, which is common knowledge. Each realization of s and ϵ will determine the contingent price p_1 of that state. Figure 7 is an example tree that depicts the underlying states and price realizations. Agent 1 believes that only the top set of states in $t = 1$, with $s = s_1$, occurs with positive probability. Agents 2 and 3 believe that only the second and the

[9] For example, Chang (2021) analyzes a network model with full-recourse collateralized debt contracts; however, the full recourse makes endogenous network formation extremely intractable.

[10] Even if the borrowers recovered their assets over the long term, the inability to recover funds in the short term caused disruption. MKM Longboat Capital Advisors closed its $1.5 billion fund partly because of frozen assets, and the chief operating officer of Olivant Ltd. committed suicide because the fund had $1.4 billion value of assets, which was believed unlikely to be recovered from the Lehman Brothers (Scott, 2014).

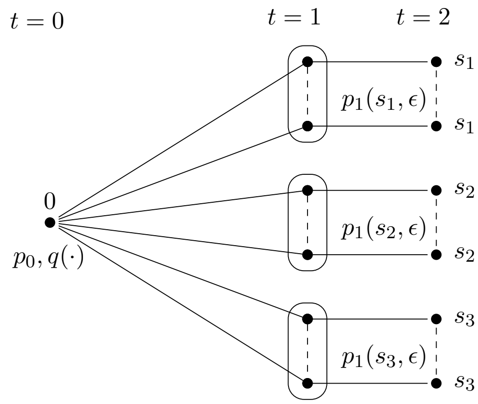

Figure 7: Tree of States and Price Realizations

third sets of states in $t = 1$ occur with positive probability, respectively. Thus, agents have their own beliefs on prices.

4. Contagion in Payment Equilibrium in Period 1

This section characterizes the contagion in payment equilibrium at $t = 1$. The payment realization in $t = 1$ shows how the given network structure and shocks affect the market price and the final wealth (and, equivalently, payoffs) of the agents. The network equilibrium in $t = 0$ is a general equilibrium with collateralized debt network formation. Because the network is formed based on the consideration of the properties of the network contagion at $t = 1$, a full characterization of the payment equilibrium is a necessary step to solve for the full model. Furthermore, the analysis of the payment equilibrium itself is also of interest related to the literature, as this attempt is one of the first to combine price contagion through endogenous asset prices with the contagion through direct debt exposures.

4.1. Preliminaries

Lender Default Loss Assumption. The case studies of the bankruptcy of Lehman Brothers and MF Global find that the lender default was more problematic for borrowers who had larger counterparty exposures, as the pool of collateral they had to recover was more complex and more likely to have a larger portion in non-segregated accounts compared with other counterparties (Fleming and Sarkar, 2014; Lleo and Ziemba, 2014; Scott, 2014; SIPC, 2016). Motivated by these studies, I assume the following properties of lender default loss functions Ψ_{ij}:

Assumption 1. *For any $i, j \in N$, Ψ_{ij} is twice differentiable in each entry of C, and*

1. *for any C, $\dfrac{\partial \Psi_{ij}}{\partial c_{ij}} = 0$ if $c_{ij} = 0$,*
2. $\Psi_i j(C) \leq c_{ij}$,

3. $\dfrac{\partial \Psi_{ij}}{\partial c_{ij}} > 0$ *and* $\dfrac{\partial^2 \Psi_{ij}}{\partial c_{ij}^2} > 0$ *if* $c_{ij} > 0$,

4. $\dfrac{\partial \Psi_{ij}}{\partial c_{ij}} > \dfrac{\partial \Psi_{ij}}{\partial c_{ik}} \geq 0$ *for any* C *with* $c_{ij}, c_{ik} > 0$,

5. $\dfrac{\partial \Psi_{ij}}{\partial c_{kj}} = \dfrac{\partial \Psi_{ij}}{\partial c_{kl}} = 0$ *for any* C *with distinct* i, j, k, l.

First, borrower j does not bear any loss if collateral exposure to i is zero. Second, the lender default cannot make repaying the debt a net loss, because borrower j can simply abandon the collateral without paying. Third, borrower j will face a larger loss, if either lender i holds a larger pool of collateral under the bankruptcy process (congestion effect) or j takes up a larger share of the same total collateral pool under the bankruptcy process (share effect). Fourth, j's losses should be more sensitive to j's own exposure to i rather than another borrower k's exposure, as the former has both the congestion and share effects whereas the latter only has the congestion effect. Fifth, borrowers' exposures to other lenders do not affect the lender default loss from i.

For example, consider $\Psi_{ij}(C) = \dfrac{c_{ij}}{\sum_k c_{ik}} \left(\dfrac{\sum_k c_{ik}}{A} \right)^2$. Even if c_{ij} remains the same, an increase in $\sum_k c_{ik}$ makes borrower j suffer more loss because of increased congestion. Also, even if $\sum_k c_{ik}$ remains the same, an increase in c_{ij} increases the share that borrower j has to bear and will increase the lender default loss for j. c_{kj} or c_{kl} does not affect Ψ_{ij}. Note that the borrower still prefers to pay in full because the lender default loss can never exceed the total gains from retrieving the collateral.

Intermediation Order. The class of possible collateralized debt networks, $\mathbb{R}_+^{n\times n} \times \mathbb{R}_+^{n\times n}$ is very large. Throughout the rest of the section, I focus on the class of networks that arises endogenously in $t = 0$, as I show in Section 5. A network is under *intermediation order* if

$$\sum_{\substack{i \in N \\ d_{ij} \geq \hat{d}}} c_{ij} \leq a_j^1 + \sum_{\substack{k \in N \\ d_{jk} \geq \hat{d}}} c_{jk} \quad \text{for any } \hat{d} \in \mathbb{R}^+ \text{ and } j \in N, \tag{6}$$

and acyclical. This condition implies that if borrower j should pay $\hat{d}$ or above in the amount of $\sum_i c_{ij}$, j has either the payments from other borrowers in the amount of $\sum_k c_{jk}$ or just the asset ownership to cover the payment. Consequentially, if the ultimate borrower fulfills the promise, the intermediary (reusing the collateral) also has enough cash to fulfill the promise to the ultimate lender. For example, consider a network of three agents with $a_1^1 = c_{21} = c_{32}$, $a_2^1 = a_3^1 = 0$, and $d_{21} < d_{32}$. This network is not under intermediation order because agent 2 is receiving less payment from agent 1 (and outright asset purchase) than agent 2 is supposed to pay agent 3. In Section 5, I show that debt networks formed under full equilibrium in $t = 0$ are under intermediation order.

4.2. Existence and Multiplicity of Payment Equilibria

First, I show that a payment equilibrium and maximum equilibrium price exist.

Proposition 1 (**Existence and Lattice Equilibrium Prices**). *For any given collateralized debt network* $(N, C, D, e^1, a^1, \epsilon, s, \Psi)$ *with* $C, D > 0$ *that is under intermediation order, there exists a payment equilibrium* (m^*, p_1^*). *Furthermore, among the set of equilibria, there always exists a maximum equilibrium* $(\overline{m}, \overline{p}_1)$, *where* $\overline{p}_1$ *is the highest equilibrium price.*

All proofs are relegated to the online appendix. The intuition of the proof is the following. The debt payment $\min\{d_{ij}, p\}$ increases as price p increases. By intermediation order, every borrower's wealth also increases as p increases because $\min\{d_{jk}, p\} - \min\{d_{ij}, p\}$ is increasing in p if $d_{jk} \geq d_{ij}$. Hence, every individual nominal wealth m_j increases in p, shown by Lemma A1 in the appendix. Because an increase in wealth also means bankruptcy is less likely, there are less lender defaults when p increases.[11] Therefore, there exists a fixed point price that clears the market.

The payment equilibrium is not unique.[12] By Proposition 1, there will always be a *maximum equilibrium* that has the least number of bankrupt agents and the highest equilibrium price for any given shocks. From now on, I will focus on the results of the *maximum equilibrium* as in Elliott et al. (2014). I assume $B(\epsilon)$ is the bankruptcy set from the maximum equilibrium price—that is, $B(\epsilon) \equiv B(\epsilon|s, \overline{p})$ from now on. With slight abuse of notation, denote $B(p) = B(\epsilon|s, \overline{p})$.

4.3. Network Contagion

Now I analyze the market price condition and its implications to financial contagion.

Suppose that price p is below s. Then, the market clearing condition, equation (3), becomes the ratio between the *remaining cash* (cash holdings and net payments to bankrupt agents minus liquidity shocks and lender default losses) $\mathcal{R}$ and the *total fire sales of the assets* that are under bankrupt agents' balance sheets (either by direct asset holdings or by collateral) $\mathcal{F}$,

$$\pi(p) \equiv \frac{\mathcal{R}(p)}{\mathcal{F}(p)} \equiv \frac{\sum_{j \notin B(\epsilon)} \left(e_j^1 - \epsilon_j - \sum_{i \in B(\epsilon)} \Psi_{ij}(C)[p - d_{ij}]^+ - \sum_{\substack{i \in B(\epsilon) \\ p \geq d_{ij}}} c_{ij} d_{ij} + \sum_{\substack{k \in B(\epsilon) \\ p \geq d_{jk}}} c_{jk} d_{jk} \right)}{\sum_{j \in B(\epsilon)} \left(a_j^1 + \sum_{\substack{k \notin B(\epsilon) \\ p < d_{jk}}} c_{jk} - \sum_{\substack{i \notin B(\epsilon) \\ p < d_{ij}}} c_{ij} \right)}, \tag{7}$$

where π is the asset price under the cash-in-the-market pricing.

The denominator $\mathcal{F}$ is non-negative and decreasing in p by the intermediation order. However, if there are no assets to be bought ($\mathcal{F}$ is zero), then the asset price will trivially be its fair value s. If there is enough cash in the market to cover the supply (fire sales) with the fair price ($\pi(s) \geq s$),

[11] Note that a low price p_1 is rather the result of more defaults than the cause of more defaults for a fixed bankruptcy set. Lender default never occurs when p_1 is sufficiently low as $\Psi_{ij}(C)[p - d_{ij}]^+$ becomes zero. Therefore, a decrease in price would rather reduce the lender default losses as long as the low price does not generate additional bankruptcy. An equilibrium is stable in terms of p_1, so that p_1 itself does not generate multiplicity.

[12] The existence of multiple equilibria implies that there could be even more instability than just focusing on the equilibrium with maximum price or welfare (Roukny et al., 2018).

then the price is also the fair value s. If the remaining cash is not sufficient to buy all the assets at fair price, then the market price will be $\pi(p) < s$, a *liquidity constrained price*.

The post-shock market clearing condition, equations (3) and (4), can be rearranged as

$$p = \begin{cases} s & \text{if } \pi(s) > s \text{ or } \mathcal{F}(p) = 0 \\ \pi(p) & \text{otherwise.} \end{cases} \tag{8}$$

The aggregate nominal wealth decreases as the price decreases or as the lender default loss increases. However, there is a feedback from the nominal wealth to the price by increased bankruptcy in (7). Such an interaction is formalized by the following proposition.

Proposition 2 (Monotone Contagion)**.** *For a given period-1 economy, suppose that (m^*, p^*) is the payment equilibrium. Then, the equilibrium wealth $(m_1^*, m_2^*, \ldots, m_n^*)$, price p^*, and the number of surviving agents $|N \backslash B^*(\epsilon)|$ are all decreasing in liquidity shock ϵ_j and lender default loss Ψ_{jk} and increasing in cash holdings e_j^1 for any $j, k \in N$.*

The negative liquidity shocks ϵ reduce available cash on the balance sheets of the agents. Lower balance sheets will cause the asset price to decline and bankruptcy of agents. Additional bankruptcy will trigger lender default, while the price decline will trigger both borrower default and valuation pressure. Both channels will cause further decline in the balance sheets of the agents, and so on. Therefore, there is an amplifying interaction between the two channels of contagion.

Define the (utilitarian ex post) social welfare as the sum of all consumption in $t = 2$ for all agents, including the outside creditors of the liquidity shocks, as

$$\sum_{j \in N} \left[e_j + a_j^1 s - \sum_{i: m_i < 0} \Psi_{ij}(C)[p_1 - d_{ij}]^+ \right]. \tag{9}$$

Note that ϵ_j for each $j \in N$ is not included in the social welfare, as payments to liquidity shocks by the agents are gains to the outside creditors, thus a zero sum. Because the asset is held by someone remaining in the market, only the lender default losses reduce the social welfare for given liquidity shocks. Proposition 2 shows the monotone comparative statics of underpricing and lender default losses. Therefore, $s - p_1$, the difference between the fundamental value and the market price of the asset, is proportional to the aggregate deadweight loss of the market. Hence, the degree of underpricing is proportional to the social welfare.

Corollary 1. *Suppose that there are two different payment equilibria with respective prices p_1 and p_1' for the same fundamental value of the asset s. If $p_1 > p_1'$, then the social welfare in the equilibrium with p_1 is greater than the social welfare in the equilibrium with p_1'.*

Thus, the expected payment equilibrium price is a measure of ex ante systemic risk of a network. In particular, $E_j[s - p_1]$ is the expected systemic risk under the subjective belief of agent $j \in N$. This notion of systemic risk follows the definition of *systemic loss* in value defined in Glasserman and Young (2016). Note that this measure focuses only on systemic risk, as the measure does not include ex ante allocative efficiency across agents for holding different assets and contracts.

Finally, comparative statics for the given economy in $t = 1$ also show that the model's contagion property makes sense. Propositions B1 and B2 as well as numerical results in the online appendix show that if the total counterparty exposure increases, agents' cash holdings increase, the likelihood of liquidity shocks decreases, or agents are more diversified, then the expected price increases, implying lower systemic risk.[13]

5. Network Formation in Period 0

This section characterizes the network formation process in a network equilibrium at $t = 0$. Agents have expectations on contagion and the resulting outcome in $t = 1$ for any given macro variables, $\tilde{p}_1$, $B(\epsilon)$, and $\Psi_{ij}(C)$ for any $i, j \in N$. In a network equilibrium, every agent maximizes expected utility for the given prices and other agents' behavior. For their portfolio decision, agents consider the expected return from a certain investment as well as the counterparty risk.

For tractability, assume $\theta_j = \theta$ for all $j \in N$, with $0 < \theta < 1$, for now. Also, assume that agent $i \in N$ believes that the asset payoff will be $s_i =\in \mathbb{R}^+$ with probability one.[14] Without loss of generality, enumerate agents by the order of their optimism as $s_1 > s_2 \cdots > s_n$. Finally, for notational simplicity, omit the + superscript over the bracket and denote $E_j[\cdot]$ as agent j's expectation conditional on non-negative nominal wealth of j.

5.1. Price and Rates in Period 0

In this subsection, the prices and interest rates are pinned down by agents' investment decisions and no-arbitrage conditions. In addition, I show that the equilibrium collateralized debt network is under intermediation order. The online appendix contains the detailed steps of deriving the equations in this subsection.

Agents solve the maximization problem (5) given their beliefs on the distribution of p_1 and $B(\epsilon)$ under shock realizations in $t = 1$. Agents have five different investment options: holding cash, buying the asset, buying the asset with leverage, lending cash to others, and lending cash with leverage. For each additional unit of cash, an agent compares the marginal returns of the five options. This return comparison will determine the interest rates and asset price.

First, I show that every agent holds a positive amount of cash in any equilibrium. Agent j's return on holding cash is the expected marginal utility of cash, $E_j[s/p_1]$. Suppose that agent j is holding zero amount of cash. Because the support of G is large enough, every agent has a positive

[13] Ibragimov et al. (2011) suggest a model with diversification of risk classes leading to systemic risk through commonality. This force is countered by the competition in the asset market and high marginal utility of cash under crisis states in my model. On the contrary, Capponi et al. (2015) show that concentration increases systemic risk when the network is *unbalancing*, which is similar to this paper because the liabilities go to one direction and increasingly toward the ultimate borrower through intermediation order.

[14] This concentrated beliefs structure—similar to that of Geerolf (2018)—is merely for tractability, and to generate gains from trade, as the main focus of this paper is not on the belief disagreements.

probability of bankruptcy. Then, there is a positive probability of p_1 being zero by (7) when agent j is holding zero cash. The marginal utility of cash is infinity under such a state, and the ex ante return on holding cash becomes infinity as well. Therefore, every agent in a network equilibrium should hold a positive amount of cash, as summarized in Lemma 1.

Lemma 1 (Positive Cash Holdings)**.** *For every agent $j \in N$, agent j's cash holding is positive, $e_j^1 > 0$, in any network equilibrium.*

This lemma shows a distinct property of the model that does not exist in other models in the financial networks literature, which often have an equilibrium in which agents choose to have strongly correlated payoffs (Allen et al., 2012; Cabrales et al., 2017; Elliott et al., 2021; Erol and Vohra, 2020; Jackson and Pernoud, 2021). The main reason of this correlated payoff structure is that agents would prefer to default whenever their counterparties are defaulting because of limited liability. An agent does not gain from surviving in a state where all other agents are defaulting on their contracts whereas the surviving agent still has to pay its obligations to defaulting agents. However, marginal utility of cash in this paper acts as an opposing force and makes agents de-correlate their payoffs from each other by holding cash. An agent in my model can purchase all the cheap assets in the market when all other agents are bankrupt.

Furthermore, the result in Lemma 1 is also important in deriving prices and rates in equilibrium. Because every agent is holding some amount of cash, the cash return $E_j\left[s/p_1\right]$ becomes the benchmark return for any other investment decision. Therefore, the cash return pins down all of the no-arbitrage conditions and greatly simplifies the problem.

Suppose that agent j is lending a positive amount without reusing the collateral in a network equilibrium. The return of lending to a contract that pays d for agent j is the expected utility of the contract payment over the cost of that contract—that is,

$$\frac{1}{q_j(d)} E_j\left[\min\left\{s, d\frac{s}{p_1}\right\}\right] = E_j\left[\frac{s}{p_1}\right],$$

and the equality holds because the return of lending should equal the return of cash for no arbitrage. This equation also represents how the price of a contract (or interest rate) is determined as

$$q_j(d) = \frac{E_j\left[\min\left\{s, d\frac{s}{p_1}\right\}\right]}{E_j\left[\frac{s}{p_1}\right]} = \frac{E_j\left[\min\left\{1, \frac{d}{p_1}\right\}\right]}{E_j\left[\frac{1}{p_1}\right]}. \tag{10}$$

Return on asset purchase without leverage is $E_j\left[s/p_0\right]$, so the return with leverage is

$$\frac{s_j}{p_0 - q_i(d)} E_j\left[\left[1 - \frac{d}{p_1}\right]^+ - \frac{\partial \Psi_{ij}(C)}{\partial c_{ij}}\left[1 - \frac{d}{p_1}\right]^+ \mathbb{1}\left\{i \in B(\epsilon)\right\}\right],$$

where agent j is borrowing cash from agent i with c_{ij} amount and promises d and $\mathbb{1}[\cdot]$ is an indicator function. Similarly, return on lending with leverage is

$$\frac{s_j}{q_j(d') - q_i(d)} E_j \left[\min\left\{1, \frac{d'}{p_1}\right\} - \min\left\{1, \frac{d}{p_1}\right\} - \frac{\partial \Psi_{ij}(C)}{\partial c_{ij}} \left[1 - \frac{d}{p_1}\right]^+ \mathbb{1}\left\{i \in B(\epsilon)\right\} \right], \tag{11}$$

where j buys (lends money) a contract with promise d'. From the return comparisons and lenders' no-arbitrage condition, an agent's individual leverage decision can be derived.

The following theorem shows that the equilibrium debt network is under intermediation order and derives the asset price and contract prices as well as the contracts traded in positive amounts in any network equilibrium. The equilibrium collateral matrix should be an acyclical network as agents borrow from more pessimistic agents, and each agent can be both borrower and lender because of return differences under subject beliefs and intermediation rents.

Theorem 1 (Intermediation Order and Contract Prices)**.** *In any network equilibrium, the following statements hold:*

1. *The collateralized debt network is under intermediation order.*

2. *For any contract with $c_{ij} > 0$, $d_{ij} = s_i$ for any $j < i \in N$.*

3. *For any $j < n$, j borrows a positive amount from $j+1$ and zero from any $i < j$.*

4. *The contract prices are*

$$q_j(d) = q_{j+1}(s_{j+1}) + \frac{E_j \left[\min\left\{1, \frac{d}{p_1}\right\} - \min\left\{1, \frac{s_{j+1}}{p_1}\right\} - \frac{\partial \Psi_{j+1,j}(C)}{\partial c_{j+1,j}} \left[1 - \frac{s_{j+1}}{p_1}\right]^+ \mathbb{1}\{j+1 \in B(\epsilon)\} \right]}{E_j\left[\frac{1}{p_1}\right]} \tag{12}$$

for any lender $j < n$ and

$$q_n(d) = \frac{E_n\left[\min\left\{1, \frac{d}{p_1}\right\}\right]}{E_n\left[\frac{1}{p_1}\right]} \tag{13}$$

for lender n.

5. *The asset price is determined by $p_0 = q_1(s_1)$ following (12).*

The intuition of the proof is the following. The most optimistic agent, agent 1, purchases the asset because agent 1 values the asset the most.[15] Also, agent 1 would like to maximize leverage and return by borrowing from another agent. Among the potential lenders, agent 2 values the collateral

[15] Agents other than agent 1—for example, say agent j—can also hold some amount of assets. In this case, agent 1 holds more cash than agent j so that the possible underpricing from larger support of p_1 for agent 1 is mitigated by being less vulnerable to liquidity shocks than others, such as agent j. Thus, $e_1^1 > e_j^1$ in such cases. This property of optimists holding more cash than pessimists can be formalized for a certain parametric region.

the most and is willing to lend more than any other agents. Therefore, agent 1 prefers to borrow from agent 2 to maximize leverage.[16] Agent 2 would like to leverage as well, and agent 2's problem is isomorphic to agent 1's problem. Debt networks following this intermediation structure naturally satisfy the intermediation order. When an agent decides to borrow from a lender, say agent i, the agent prefers to promise the maximum possible amount—that is, s_i—to maximize leverage. The price equations come from equating the return in (11) with cash return because of Lemma 1.

The results in Theorem 1 also have implications on the pattern of haircuts and interest rates that match recent empirical evidence in the literature (Baklanova et al., 2019; Jank et al., 2021). First, there can be multiple haircuts for the same collateral asset. Second, high levels of reuse would lead to wider dispersion in rates. Third, the model could explain the weak relationship between haircuts and interest rates as shown by the following corollary of Theorem 1.

Corollary 2. *In a network equilibrium, there can be multiple haircuts for the same collateral asset. Also, the relationship between haircuts and interest rates may not be strictly negative.*

This result implies that my model with reuse of collateral and lender default can replicate empirical observations unable to be replicated by existing models focusing on borrower default. Baklanova et al. (2019) show that multiple haircuts are used for the same (CUSIP-level) collateral security. Also, Jank et al. (2021) show that high levels of reuse would lead to higher volatility of rates. Endogenous reuse of collateral in my model generates multiple haircuts for the same asset, and the dispersion in rates increases when the level of reuse of collateral increases. Also, Baklanova et al. (2019) find that the relationship between haircuts and rates is not as significant as predicted by the theoretical models based on borrower default. The existence of lender default loss in my model can distort the interest rates across different haircuts. Corollary 2 shows that these empirical patterns are natural results of my model that incorporates reuse of collateral and lender default.

By Theorem 1, I can focus on the class of debt networks under intermediation order. To be precise, the equilibrium contract matrix D^* is a lower triangle matrix with $d^*_{ij} = s_i$ for any $i > j$ for $j < n-1$ and $d^*_{ij} = 0$ otherwise. Also, Theorem 1 greatly simplifies the agent's optimization problem because the problem becomes determining the optimal weights of collateral exposure to different borrowers and lenders (for the fixed contract matrix D^*).

5.2. Equilibrium Allocation in Period 0

Given the prices in $t = 0$, the remaining parts of the network equilibrium are the amount of cash holdings and the amount traded for each contract. The first tradeoff for each agent is the tradeoff between leverage and counterparty risk. The second tradeoff is the tradeoff between holding cash and purchasing collateralized debt (or the asset). These individual tradeoffs determine each agent's optimal portfolio while the equilibrium prices clear the market.

[16] Geerolf (2018) and Gong and Phelan (2020) also show that agents maximize leverage at each step of the intermediation chain.

Theorem 2 (Existence and Characterization of Network Equilibrium).
For a given economy $(N, (s_j, \theta_j, e^0)_{j\in N}, A, \Psi, G)$ *and maximum equilibrium selection rule, there exists a network equilibrium* $(C, D, e^1, a^1, p_0, \tilde{p}_1, q)$*, which is characterized as follows:*

1. *For any* $i, j \in N, i \neq j$, $d_{ij} = s_i$ *and* (C, D) *is under intermediation order.*
2. *For any* $j, i \in N$ *and* $j \leq i$, $c_{ji} = 0$.
3. *For any counterparties* i, k *of* j *with* $c_{ij} > 0, c_{kj} > 0$,

$$\begin{aligned}&\frac{s_j}{q(s_j)-q(s_i)}E_j\left[\min\left\{1,\frac{s_j}{p_1}\right\}-\min\left\{1,\frac{s_i}{p_1}\right\}-\frac{\partial\Psi_{ij}(C)}{\partial c_{ij}}\left[1-\frac{s_i}{p_1}\right]^+\mathbb{1}\left\{i\in B(\epsilon)\right\}\right]\\=&\frac{s_j}{q(s_j)-q(s_k)}E_j\left[\min\left\{1,\frac{s_j}{p_1}\right\}-\min\left\{1,\frac{s_k}{p_1}\right\}-\frac{\partial\Psi_{kj}(C)}{\partial c_{kj}}\left[1-\frac{s_k}{p_1}\right]^+\mathbb{1}\left\{k\in B(\epsilon)\right\}\right].\end{aligned}$$

4. *Cash holdings of each agent* j *are* $e_j^1 = e_j^0 + \sum_{i\in N} c_{ij}q(s_i) - \sum_{k\in N} c_{jk}q(s_j) - a_j^1 p_0$.
5. *Contract prices* q *and the asset price* p_0 *at* $t = 0$ *are determined by Theorem 1.*
6. *The price of the asset at* $t = 1$, $\tilde{p}_1$ *is determined by payment equilibrium for* (C, D).

Theorem 2 contains two main implications. First, statement 3 of Theorem 2 suggests the first main mechanism of network formation—the tradeoff between leverage and counterparty risk. If the lender counterparty risk is negligible (small Ψ or θ), a single-chain network is formed. Even if $c_{j+1,j}$ is large, the return of borrowing from $j + 1$ is still greater than the return of borrowing from $l > j + 1$, as the increase in counterparty risk is small. Agents are not concerned about diversifying their counterparties and choose the most profitable counterparty—that is, the most optimistic agent after themselves—concentrating all their collateral exposure on that counterparty.

However, if the lender counterparty risk is non-negligible, then a multi-chain network is formed in equilibrium. Agent j borrows not only from $j + 1$, but also from $j + 2$. Agents would diversify their counterparties and would like to link with several levels down of optimism. However, this lower counterparty risk comes at the cost of lower leverage (a higher haircut). This network formation mechanism, the tradeoff between leverage and counterparty risk, makes the intermediation pattern distinct from other models in the literature.

The second implication of Theorem 2 is leverage stacking through the lending chain. An increase in $q(s_n)$ increases all the subsequent contract prices through the recursive equation (12), which implies that the lending amount increases. Therefore, leverage in the lending chain has a multiplier effect due to reuse of collateral, which Gottardi et al. (2019) have also examined. A distinct feature from Theorem 2 is that different levels in the lending chain have different multiplier effects. An increase in s_n will have a larger effect than an increase in s_2, as agent n's lending stacks $n - 1$ times through the lending chain through equation (12). A real-world implication could be that the increase in the confidence of the ultimate lender can lead to a huge increase in asset prices through this multiplier effect.

Finally, note that infinitely many debt contracts are still available for trade in the market. The price of a contract with any arbitrary d for a given counterparty is already determined in the market by equation (12). However, only a few of those contracts are actually traded in positive amount in equilibrium as in Geanakoplos (1997).

The next result is on the effect of diversification in a network equilibrium. Diversification of lenders creates positive externalities to other agents by making the overall network safer. Thus, systemic risk (the expected asset price) under any agent's belief decreases (increases). Because agents do not take this decrease in systemic risk into account, the degree of diversification is always less than the optimal degree in the economy, and the equilibrium could be inefficient.[17] This result is similar to the counterparty concentration externality in Frei et al. (2021).

Before I proceed to the diversification result, I define the diversification of counterparties. For a debt network (C, D), define $\tilde{C}$ as a *diversification of agent* j from C, if

1. $\sum_{i \in N} \Psi_{ij}(C)\omega_{ij}(d_{ij}; C) > \sum_{i \in N} \Psi_{ij}(\tilde{C})\omega_{ij}(d_{ij}; C)$,
2. $\sum_{i \in N} c_{ij} \geq \sum_{i \in N} \tilde{c}_{ij}$,
3. $c_{ik} \geq \tilde{c}_{ik}$ for all $i, k \in N$ with $k \neq j$, and
4. $(\tilde{C}, D)$ is under intermediation order.

This diversification of agent j from a given collateral matrix implies that agent j has more diversified counterparties than the original network in either intensive or extensive margins.

Proposition 3 (Diversification and Systemic Risk)**.** *Suppose that* $(C, D, e^1, a^1, p_0, \tilde{p}_1, q)$ *is a network equilibrium. Then, there exists an allocation with higher expected asset prices and lower systemic risk under any agent's belief that is a diversification of agent* j *from* C *and paying respective contract prices* q *for the change in collateral matrix.*

Unlike the unsecured debt network models in the literature, the too-interconnected-to-fail problem is not an issue in this model because all the debt connections are collateralized. Weight of the flows (exposures) also matters in my model, and there are limits to the total weight for each agent by the collateral constraints. Therefore, more links would always imply more diversified and lower flow volume of collateral, which would always lead to lowering the systemic risk in my model.

The next result is the endogenous market reaction to the change in counterparty risk. As the counterparty risk increases, agents diversify their counterparties more, and the overall leverage and debt decrease by the tradeoff between counterparty risk and leverage shown in statement 3 of Theorem 2. The intuition for this result is the following. Agents prefer to hold more cash if severe liquidity shocks are more likely. Then, agents are also willing to lend less, and contract prices decrease. Thus, the overall debt decreases not only by the decrease in leverage from lender

[17] In Proposition B3 in the online appendix, I show that there always exists an allocation that Pareto dominates the equilibrium allocation under an additional assumption.

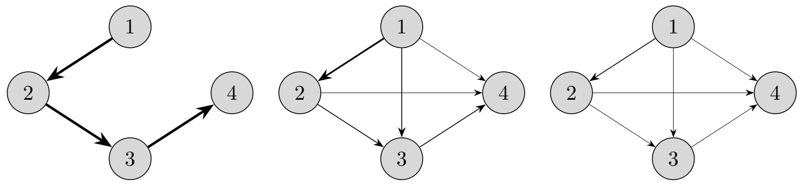

Figure 8: Comparison of Endogenous Networks – no risk, moderate risk, and significant risk

	No risk	Moderate risk	Significant risk
Liquidity shock probability (θ)	0	0.4	0.8
Pr(Bankruptcy)	0%	9.6%	25.4%
Leverage	10	2.0766	1.7411
Collateral multiplier	3	1.6870	1.4149
Volume of debt	2400	756	431
Number of links	3	6	6

Table 1: Comparison of Endogenous Networks

diversification, but also by the decrease in asset and contract prices. To complete the full comparative statics, consider the marketwide effect of the change in a borrowing pattern. Because the diversification of lenders will increase the expected asset price as in Proposition 3, the asset price in $t = 1$ is more likely to exceed the debt amount. Then, they are more likely to suffer the lender default loss and have even more incentives to diversify.

Theorem 3 (Network Change under Changes in Counterparty Risks)**.** *If the counterparty risks in the economy become greater as θ_j increases, then agents diversify their counterparty exposures more, the asset price decreases, the average leverage decreases, the reuse of collateral decreases, and the average number of counterparties increases.*

The results of Theorem 3 are consistent with the empirical facts. As Singh (2017) documented, the velocity (reuse) of collateral decreased from 3 to 2.4 right after the bankruptcy of the Lehman Brothers, and the average leverage in the over-the-counter (OTC) market also went down. In the wake of Bear Stearns' demise, hedge funds, which used to have business with only one prime broker, had increasingly used multiple prime brokers to mitigate counterparty risk (Scott, 2014). After the Lehman bankruptcy, hedge funds increased the number of prime brokers they work with even further and the prime brokerage market became much more competitive—which translates into lower intermediation rents under Theorem 3—after the crisis (Eren, 2015; Sinclair, 2020). Moreover, the average number of linkages between financial institutions increased about 30 percent over the four years after the Lehman bankruptcy, and hedge funds diversified their portfolio of counterparties almost immediately after the Lehman bankruptcy (Craig and Von Peter, 2014; Eren, 2015; Sinclair, 2020).

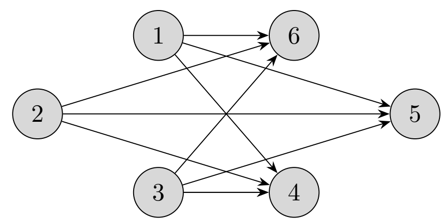

Figure 9: Single leverage complete bi-partite network

Numerical examples in figure 8 and table 1 illustrate the comparative statics in Theorem 3. Figure 8 represents the collateral flow of promises of no-risk, moderate risk, and significant risk cases, respectively. Each numbered node represents the agent, and the arrow link represents the direction and size of exposure. As risk increases, equilibrium network changes from a single-chain network with large collateral flows to a multi-chain network with smaller collateral flows. As the liquidity shock and counterparty risk become more relevant, the probability of bankruptcy increases. The leverage of agent 1 decreases by a huge margin, and the collateral multiplier decreases as agents diversify their counterparties, which reduces the reuse of collateral. The number of links increases because of diversification and the total volume of debt decreases.

5.3. Exogenous Leverage or Network

Exogenous leverage models completely miss the main results in this paper. If an exogenously given leverage (or haircut) is fixed as d per unit of collateral with price $q(d)$, then agents will be divided into two groups—buyers (borrowers) and sellers (lenders) of the asset. Then, there is no tradeoff between leverage and counterparty risk because there is only one contract. Agents will fully diversify their counterparties. Thus, a complete bi-partite network as in figure 9 is the equilibrium network under exogenous leverage. This outcome is in line with the actual network structure of the triparty repo market, which is a bi-partite network with a fixed contract terms for a large set of collateral (Copeland et al., 2014). Because agents are already diversifying fully, a policy intervention that pools counterparty risks has zero effect on network formation.

Proposition 4. *If only one contract d is available in the market, then the equilibrium network is a complete bi-partite network.*

Furthermore, exogenous network models would also miss the full picture of a market condition change or a policy intervention. Because the debt relationships are already fixed and the debt matrix is endogenously fixed as D^*, the only factor that is changing is the contract prices $q(d)$. Although a policy change such as mandating central clearing would increase the contract prices, the overall counterparty exposures would remain the same as the network is fixed.

These results show that simultaneously endogenizing leverage, asset prices, and network formation is necessary to generate the change in the tradeoff between counterparty risk and leverage. Therefore, this paper finds a novel feature of endogenous change in network structure that has not been explored in the literature.

6. Discussion

6.1. Allocative Efficiency versus Systemic Risk

The ex ante social welfare in the model comprises two major parts: the allocative efficiency and systemic risk, on which this paper focuses. The allocative efficiency is maximized under a single-chain network (maximum reuse of collateral) because each agent effectively buys (bets) the tranche of the asset in which the agent believes. However, a single-chain network also maximizes systemic risk by the concentration of network and the maximum amount of leverage. While the analysis in this paper focused exclusively on the systemic risk side, the overall social welfare should depend on the balance between the two (Gofman, 2017). Therefore, the main contribution of this paper should be considered as identifying a new mechanism of how the systemic risk changes when the underlying conditions change the endogenous network structure.

6.2. Intermediation Spread with Additional Heterogeneity

The analysis so far has assumed homogeneous endowments and liquidity shock distributions. The setup of the model generates varying intermediation spreads and relative position in the network structure based solely on the heterogeneous beliefs. In reality, this intermediation spread and network structure could also depend heavily on other dimensions of heterogeneity, such as size of each agent and idiosyncratic liquidity shock distribution.

The model in this paper can be extended to allow heterogeneous endowments and shocks. If an agent has large endowments, then the agent can play a central role in the intermediation structure. In particular, the agent can become a core intermediary that intermediates trades across different agents by assuming the counterparty risk while charging higher spreads. This setup can potentially replicate the spread structure typically observed in the data—a spread between inter-dealer rates and triparty rates in the repo markets. A similar analysis is possible if an agent has a lower probability of receiving liquidity shocks. Even more degree of freedom is possible by allowing heterogeneous costs for each pair as $\Psi_{ij}(C)$. Although this dimension is not fully exploited in the previous sections, such a heterogeneous cost structure would be crucial in estimating the parameters empirically and replicating the core-periphery structure in OTC markets as in Craig and Ma (2021).

Moreover, the haircut for a hedge fund's contract is typically greater than the haircut for a dealer's contract when they borrow from money market mutual funds (Baklanova et al., 2019). Introducing size and cost heterogeneity can attain the haircut differences. If a dealer is much larger than its counterparties (as the dominant dealers observed in the data), then the dealer may be able to trade with other agents under a much lower haircut. More formal analysis on the possible heterogeneity is left for future extensions.

6.3. Policy Implications

The results suggest that a measure of systemic risk due to interconnectedness has to include both reuse of collateral and leverage. A combination of low level of reuse and low leverage implies low systemic risk. An increase in either implies higher vulnerabilities of the market. An increase in both reuse and leverage would be much more concerning, as they could amplify the price swings as discussed in Section 4. Therefore, monitoring both the leverage and interconnectedness of the market is important in accurately assessing systemic risk.

Theorem 3 suggests that a policy change mitigating counterparty losses can have a side effect of exacerbating the diversification externality problem by the endogenous changes in the network. Such a policy at a glance may seem to reduce systemic risk as individual counterparty risk for each agent decreases. However, the tradeoff between counterparty risk and leverage disappears when individual counterparty risk is insulated. Agents will form a single-chain network (or a network with less diversification), which maximizes leverage, reuse of collateral, and systemic risk.

For example, mandating central clearing by a CCP was one of the key elements of the financial system reforms addressed by central banks and financial authorities after the financial crisis in 2008. The spikes in the repo market in September 2019 and the COVID-19 crisis in March 2020 have revitalized the discussion on CCPs (Duffie, 2020). CCPs can decrease systemic risk by insulating clearing members from counterparty risks by novation, reducing counterparty exposures significantly by netting, and efficient collateral management. However, there could be a hidden side effect of an increase in systemic risk by endogenous network responses highlighted in this paper.

Indirect ways of addressing the risks related to concentration and reuse of collateral are already active in the collateralized debt markets. For example, there are single counterparty exposure limits, large exposure caps, and CCPs' initial margin (collateral) management such as concentration margin add-ons. These measures will mitigate the extreme form of concentration and reuse of collateral. Various forms of leverage regulations would help mitigate the systemic risks coming from price swings and liquidity shocks as well.

A more direct regulation to solve for the diversification externality problem could be introducing a relevant leverage ratio restriction. A slight modification of the Basel III supplementary leverage ratio, which is effectively a tax on intermediation activity that is proportional to the size of an intermediary's balance sheet, can be used as

$$\frac{\text{Tier 1 Capital}}{(c_{1i}^2 + c_{2i}^2 + \cdots + c_{ni}^2) \times \text{Total Leverage Exposure}},$$

and risk externality can be included as weights of counterparty exposure in the denominator. These restrictions provide marginal incentives to diversify, internalize second-order defaults, and maintain borrower or lender discipline of agents. Such a marginal adjustment is difficult to implement in existing measures such as single counterparty exposure limits, large exposure cap, and global systemically important bank capital surcharge in interconnectedness.

A supplementary policy is liquidity injection to the agent under distress according to its effect on the system, as in Demange (2016). This injection or bail-out idea also faces side effects from moral hazard in terms of network formation (Leitner, 2005; Erol and Vohra, 2020).

7. Conclusion

I constructed a general equilibrium model with collateral featuring endogenous leverage, endogenous price, and endogenous network formation, which is the first attempt in the literature. The model bridges the theories of financial networks and general equilibrium with collateral. Collateralized debt has an additional amplification channel through asset price risk—the price channel—on top of the balance sheet risk through the debt network—the counterparty channel.

Borrowers diversify their portfolio of lenders because of the possibility of lender default. However, lower counterparty risk comes at the cost of lower leverage. Positive externalities arise from diversification because it reduces not only the individual counterparty risk, but also the systemic risk, by limiting the propagation of shocks and price volatility. The key externalities here, arising from the tradeoff between counterparty risk and leverage, are absent in models with exogenous leverage or networks. This network formation mechanism shows that a policy that mitigates individual counterparty risk could have negative side effects by distorting the tradeoff between counterparty risk and leverage.

The model also generates predictions that fill in the gap between the empirical data and the existing models in the literature. Greater counterparty risk induces agents to diversify more, which lowers leverage and reuse of collateral, and increases the number of links. Moreover, the model explains how there could be multiple haircuts for the same asset used as collateral and why the relationship between haircuts and interest rates may not be strictly negative in contrast to the models focusing only on borrower default.

References

Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi (2015): "Systemic Risk and Stability in Financial Networks," *American Economic Review*, 105, 564–608.

Acharya, V. and A. Bisin (2014): "Counterparty Risk Externality: Centralized versus Over-the-counter Markets," *Journal of Economic Theory*, 149, 153–182.

Allen, F., A. Babus, and E. Carletti (2012): "Asset Commonality, Debt Maturity and Systemic Risk," *Journal of Financial Economics*, 104, 519–534.

Allen, F. and D. Gale (2000): "Financial Contagion," *Journal of Political Economy*, 108, 1–33.

Babus, A. (2016): "The Formation of Financial Networks," *The RAND Journal of Economics*, 47, 239–272.

Babus, A. and T.-W. Hu (2017): "Endogenous Intermediation in Over-the-Counter Markets," *Journal of Financial Economics*, 125, 200–215.

Babus, A. and P. Kondor (2018): "Trading and Information Diffusion in Over-the-Counter Markets," *Econometrica*, 86, 1727–1769.

Baklanova, V., C. Caglio, M. Cipriani, and A. Copeland (2019): "The Use of Collateral in Bilateral Repurchase and Securities Lending Agreements," *Review of Economic Dynamics*, 33, 228–249.

Bignon, V. and G. Vuillemey (2020): "The Failure of a Clearinghouse: Empirical Evidence," *Review of Finance*, 24, 99–128.

Brusco, S. and F. Castiglionesi (2007): "Liquidity Coinsurance, Moral Hazard, and Financial Contagion," *The Journal of Finance*, 62, 2275–2302.

Cabrales, A., P. Gottardi, and F. Vega-Redondo (2017): "Risk Sharing and Contagion in Networks," *The Review of Financial Studies*, 30, 3086–3127.

Capponi, A., P.-C. Chen, and D. D. Yao (2015): "Liability Concentration and Systemic Losses in Financial Networks," *Operations Research*, 64, 1121–1134.

Capponi, A. and M. Larsson (2015): "Price Contagion through Balance Sheet Linkages," *The Review of Asset Pricing Studies*, 5, 227–253.

Chang, B. and S. Zhang (2021): "Endogenous Market Making and Network Formation," *Available at SSRN: https://ssrn.com/abstract=2600242*.

Chang, J.-W. (2021): "Contagion in Debt and Collateral Markets," *Working Paper*.

Cifuentes, R., G. Ferrucci, and H. S. Shin (2005): "Liquidity Risk and Contagion," *Journal of the European Economic Association*, 3, 556–566.

Copeland, A., A. Martin, and M. Walker (2014): "Repo Runs: Evidence from the Tri-Party Repo Market," *The Journal of Finance*, 69, 2343–2380.

Craig, B. and G. Von Peter (2014): "Interbank Tiering and Money Center Banks," *Journal of Financial Intermediation*, 23, 322–347.

Craig, B. R. and Y. Ma (2021): "Intermediation in the Interbank Lending Market," *Journal of Financial Economics*, forthcoming.

Demange, G. (2016): "Contagion in Financial Networks: A Threat Index," *Management Science*, 64, 955–970.

Di Maggio, M. and A. Tahbaz-Salehi (2015): "Collateral Shortages and Intermediation Networks," *Working Paper*.

Diamond, D. W. and P. H. Dybvig (1983): "Bank Runs, Deposit Insurance, and Liquidity," *The Journal of Political Economy*, 401–419.

Duarte, F. and T. M. Eisenbach (2021): "Fire-sale Spillovers and Systemic Risk," *Journal of Finance*, 76, 1251–1294.

Duarte, F. and C. Jones (2017): "Empirical Network Contagion for U.S. Financial Institutions," *Staff Report, No. 826, Federal Reserve Bank of New York.*

Duffie, D. (2020): "Still the World's Safe Haven?" *Redesigning the US Treasury market after the COVID-19 crisis', Hutchins Center on Fiscal and Monetary Policy at Brookings, available online at https://www. brookings. edu/research/still-the-worlds-safe-haven.*

Eisenberg, L. and T. H. Noe (2001): "Systemic Risk in Financial Systems," *Management Science*, 47, 236–249.

Elliott, M., B. Golub, and M. O. Jackson (2014): "Financial Networks and Contagion," *American Economic Review*, 104, 3115–53.

Elliott, M., J. Hazell, and C.-P. Georg (2021): "Systemic Risk Shifting in Financial Networks," *Journal of Economic Theory*, 191, 105–157.

Eren, E. (2015): "Matching Prime Brokers and Hedge Funds," *Available at SSRN 2686777.*

Erol, S. and R. Vohra (2020): "Network Formation and Systemic Risk," *Available at SSRN: https://ssrn.com/abstract=2546310.*

Farboodi, M. (2017): "Intermediation and Voluntary Exposure to Counterparty Risk," *Available at SSRN 2535900.*

Financial Stability Board (2017): "Transforming Shadow Banking into Resilient Market-based Finance: Re-hypothecation and collateral re-use: Potential financial stability issues, market evolution and regulatory approaches," *https://www.fsb.org/wp-content/uploads/Re-hypothecation-and-collateral-re-use.pdf.*

Fleming, M. and A. Sarkar (2014): "The Failure Resolution of Lehman Brothers," *Economic Policy Review*, 20, 175–206.

Fostel, A. and J. Geanakoplos (2015): "Leverage and Default in Binomial Economies: a Complete Characterization," *Econometrica*, 83, 2191–2229.

Frei, C., A. Capponi, and C. Brunetti (2021): "Counterparty Risk in Over-the-Counter Markets," *Journal of Financial and Quantitative Analysis*, forthcoming.

Freixas, X., B. M. Parigi, and J.-C. Rochet (2000): "Systemic Risk, Interbank Relations, and Liquidity Provision by the Central Bank," *Journal of Money, Credit and Banking*, 32, 611–638.

Fuhrer, L. M., B. Guggenheim, and S. Schumacher (2016): "Re-Use of Collateral in the Repo Market," *Journal of Money, Credit and Banking*, 48, 1169–1193.

Gai, P., A. Haldane, and S. Kapadia (2011): "Complexity, Concentration and Contagion," *Journal of Monetary Economics*, 58, 453–470.

Geanakoplos, J. (1997): "Promises Promises," in *The Economy as an Evolving Complex System II*, ed. by S. D. Brian Arthur and D. Lane, Reading, MA: Addison-Wesley, 285–320.

——— (2003): "Liquidity, Default, and Crashes Endogenous Contracts in General Equilibrium," in *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress*, Cambridge University Press, vol. 2, 170.

——— (2010): "The Leverage Cycle," *NBER Macroeconomics Annual*, 24, 1–65.

Geerolf, F. (2018): "Leverage and Disagreement," *Working Paper*.

Ghamami, S., P. Glasserman, and H. P. Young (2021): "Collateralized networks," *Management Science*.

Glasserman, P. and H. P. Young (2016): "Contagion in Financial Networks," *Journal of Economic Literature*, 54, 779–831.

Gofman, M. (2017): "Efficiency and Stability of a Financial Architecture with Too-interconnected-to-fail Institutions," *Journal of Financial Economics*, 124, 113–146.

Gong, F. and G. Phelan (2020): "Debt collateralization, capital structure, and maximal leverage," *Economic Theory*, 70, 579–605.

Gorton, G. and A. Metrick (2012): "Securitized Banking and the Run on Repo," *Journal of Financial Economics*, 104, 425–451.

Gottardi, P. and F. Kubler (2015): "Dynamic competitive economies with complete markets and collateral constraints," *The Review of Economic Studies*, 82, 1119–1153.

Gottardi, P., V. Maurin, and C. Monnet (2019): "A Theory of Repurchase Agreements, Collateral Re-Use, and Repo Intermediation," *Review of Economic Dynamics*, 33, 30–56.

Ibragimov, R., D. Jaffee, and J. Walden (2011): "Diversification Disasters," *Journal of Financial Economics*, 99, 333–348.

Infante, S. (2019): "Liquidity Windfalls: The Consequences of Repo Rehypothecation," *Journal of Financial Economics*.

Infante, S., C. Press, and J. Strauss (2018): "The Ins and Outs of Collateral Re-use," *FEDS Notes*, https://doi.org/10.17016/2380–7172.2298.

Infante, S. and Z. Saravay (2020): "What Drives U.S. Treasury Re-use?" *FEDS Working Paper No. 2020-103.*

Infante, S. and A. Vardoulakis (2021): "Collateral Runs," *The Review of Financial Studies*, 34, 2949–2992.

Jackson, M. O. and A. Pernoud (2021): "Investment Incentives and Regulation in Financial Networks," *Available at SSRN: https://ssrn.com/abstract=3311839.*

Jank, S., E. Moench, and M. Schneider (2021): "Safe Asset Shortage and Collateral Re-use," *Working Paper.*

Leitner, Y. (2005): "Financial Networks: Contagion, Commitment, and Private Sector Bailouts," *The Journal of Finance*, 60, 2925–2953.

Lleo, S. and W. Ziemba (2014): "How to Lose Money in the Financial Markets: Examples from the Recent Financial Crisis," *Available at SSRN: https://ssrn.com/abstract=2478252.*

Martin, A., D. Skeie, and E.-L. Von Thadden (2014): "Repo Runs," *Review of Financial Studies*, 27, 957–989.

Park, H. and C. M. Kahn (2019): "Collateral, Rehypothecation, and Efficiency," *Journal of Financial Intermediation*, 39, 34–46.

Rochet, J.-C. and J. Tirole (1996): "Interbank Lending and Systemic Risk," *Journal of Money, Credit and Banking*, 28, 733–762.

Roukny, T., S. Battiston, and J. E. Stiglitz (2018): "Interconnectedness as a Source of Uncertainty in Systemic Risk," *Journal of Financial Stability*, 35, 93–106.

Scott, H. S. (2014): "Interconnectedness and Contagion - Financial Panics and the Crisis of 2008," *Available at SSRN 2178475.*

Simsek, A. (2013): "Belief Disagreements and Collateral Constraints," *Econometrica*, 81, 1–53.

Sinclair, A. (2020): "Do Prime Brokers Intermediate Capital?" *Working paper.*

Singh, M. M. (2017): *Collateral Reuse and Balance Sheet Space*, International Monetary Fund.

SIPC (2016): "SIPC: Customers See Full Assets Restored as MF Global Liquidation Ends," *Press Release.*

Upper, C. (2011): "Simulation Methods to Assess the Danger of Contagion in Interbank Markets," *Journal of Financial Stability*, 7, 111–125.

Zawadowski, A. (2013): "Entangled Financial Systems," *The Review of Financial Studies*, 26, 1291–1323.