Moldy Lemons and Market Shutdowns*

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Abstract

This paper studies conditions under which small changes in fundamentals cause competitive markets to shutdown due to adverse selection in a non-exclusive contracting environment. We first show that markets with non-exclusive competition are generally robust to small changes in the risk profile of agents (moldy lemons), which stands in stark contrast to exclusive contracting environments where trade may suddenly cease. We extend the environment to include outside options and establish a necessary and sufficient condition for trade to breakdown, and show that the entry of moldy lemons can trigger what we call exit cascades and a breakdown in market trades. Finally, we show that more precise information about agents' types makes markets more prone to exit cascades. The model is general, does not rely on institutional details or structure, and thus applicable to many different markets and contexts.

JEL Classification: D52, D53, D82, E44, G32
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1 Introduction

How can a market that functions well in normal times suddenly collapse under stress? Throughout history—from banking panics in the national banking era to the dry-up of asset-backed securities markets (for example, see Figures 1 and 2) and freezing of interbank markets in the Global Financial Crisis—one common answer is adverse selection (Beltran et al., 2017; Calomiris and Gorton, 1991; Mishkin, 1999; Ivashina and Scharfstein, 2010; Covitz et al., 2013; Foley-Fisher et al., 2020). Canonical models that feature market failure due to adverse selection beginning with Akerlof (1970) and Rothschild and Stiglitz (1976) assume exclusive contracting between agents. However, in practice, most markets are characterized by nonexclusive contracting where agents are generally free to trade and contract simultaneously with multiple counterparties (for example, collateralized debt and loan obligations (CDOs and CLOs), derivatives, over-the-counter (OTC), capital, and insurance markets). The inability to monitor a counterparty's trades was certainly a major factor behind the 2021 collapse of family office Archaegos and the bail out of AIG in 2008 due to its credit derivatives positions.

This paper studies conditions under which trade in markets subject to adverse selection and non-exclusive competition break down due to a small change in underlying fundamentals. To make progress on this issue, we build on the recent work by Attar et al. (2021) who develop a general model of non-anonymous trade where agents' types are private information and they are free to contract with as many counterparties as they wish. Importantly, the equilibrium allocation in their framework always exists and is unique, which circumvents the non-existence issues that plague adverse selection economies (Attar et al., 2014; Rothschild and Stiglitz, 1976). Furthermore, the equilibrium allocation features multiple contracts trading in equilibrium at different prices and quantities rather than a single pooling contract as in Akerlof (1970) and Attar et al. (2011).

We believe the Attar et al. (2021) allocation closely resembles real world settings in insurance markets where insurance companies know with whom they contract, but policy holders are free to obtain multiple policies from different insurers. Similarly, in OTC derivative markets, clients trade with a relatively small set of dealers with close client relationships. However, client trades across dealers are unobserved from the dealer perspective. Hence, trade is non-exclusive across suppliers and non-anonymous with a given supplier. In both settings, the more contracts clients purchase, which may be a standardized products, the higher the premium or price.

Our main results can be stated as follows: In the baseline model of Attar et al. (2021), the presence of the worst type of agent (a moldy lemon) causes trade to unravel only if their mass is sufficiently large. By contrast, when agents have outside options or reservation utilities, a small mass of moldy lemons can lead to a large cascade of exits. Hence, we uncover a novel channel and conditions under which a small change in fundamentals amplifies into a broader market collapse. Moreover, we show that markets are more susceptible to unraveling as more precise information about agents' types is revealed. Thus, we show in a simple framework how "information events" such as information production can generate a breakdown in trade.

We derive these results in a general and flexible model of perfectly competitive trade subject to adverse selection that allows for non-exclusive contracting between buyers and sellers following Attar et al. (2021). This non-exclusive contracting framework imposes little structure on prices and quantities and abstracts away from complicated model structures or restrictions backed by institutional details (e.g. information production in Dang et al. (2020), dynamics of collateral and reputation in Chari et al. (2014), or dynamic self-fulfilling market freezes in Malherbe (2014); Asriyan et al. (2019)) that are generally needed to generate market shutdowns through small changes in underlying fundamentals; all we require is a standard single-crossing property on preferences and a monotonicity condition on costs. Together, these conditions imply that types more willing to trade larger amounts are more costly to serve. Hence, there is weak-adverse selection.

Following Attar et al. (2021), we characterize a market shut down or unraveling of trade when an active market becomes inactive and "entry-proof." An inactive market is one for which the no-trade point dominates trade in the market; markets are entry-proof when the willingness of each agent to trade at the no-trade point does not exceed the cost to serve all types that will enter the market. Finally, the cost to serve the market is given by the upper-tail conditional expectation of the unit cost of all agents expected to trade.

We extend the environment of Attar et al. (2021) by considering an agent that is the worst type among all possible types—that is, extending the lower tail of the distribution of types.[1] We call this agent a "moldy lemon." In an investment or trading environment, moldy lemons are agents whose project or asset has the lowest expected payout. Thus, our main research question becomes: Does a shift in underlying fundamentals that create a small mass of moldy lemons cause the market to shutdown?

Before turning to the impact of the entry of moldy lemons, it is useful to understand how allocations in the general non-exclusive contracting economy of Attar et al. (2021) are characterized. In technical terms, the allocations in this environment are recursive layers along a convex-market tariff. Along the first layer all types trade, and the contracts are

[1]To be precise, our results go through for any general shift in the distribution that changes both the unconditional and conditional means. By contrast, shifts in variance or second order stochastic dominant changes are not sufficient to preserve our results.

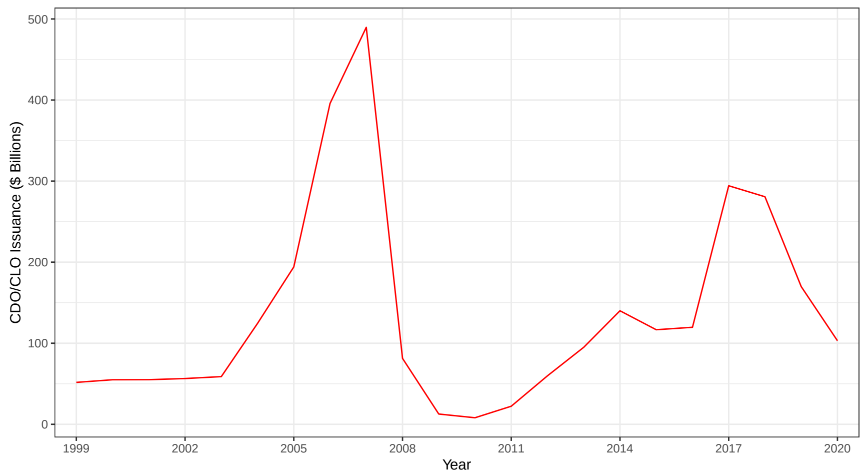

Figure 1: Issuance of Collateralized Debt Obligations and Collateralized Loan Obligations

Source: Securities Industry and Financial Markets Association.

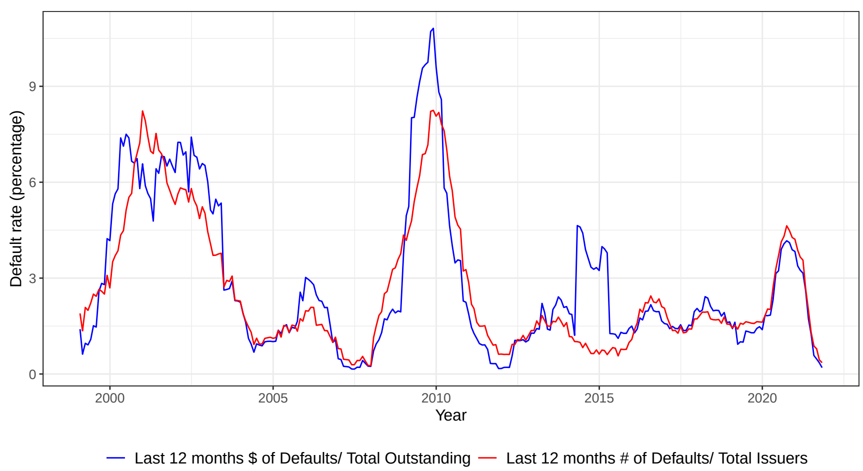

Figure 2: U.S. leveraged loan default rate

Note: CDO and CLO issuance plummets in times of crises, but the default rate of the underlying securities–leveraged loans–modestly increases.
Source: S&P Global Market Intelligence.

traded at the lowest price equal to the expected cost to serve all types. The quantity traded equals the demand of the lowest (best in suppliers' perspective) type. Along the second layer, the lowest type does not trade, and the price is equal to the expected cost of serving all types except the first type. Along this layer, the quantity each agent—other than the lowest type—trades is the residual demand of the second type. This structure continues until the final layer meets the residual demand of the highest type for whom sellers break even to serve. Hence, all types except the lowest type generally combine layers along the market tariff to arrive at an aggregate level of trade, which is prohibited by assumption in exclusive contracting environments. Note that the unit price increases across these layers, forming a convex-market tariff for the entire market allocation.[2]

Our first result stated in Theorem 1 is that moldy lemons change the equilibrium quantities traded in a smooth fashion. This implies that markets shut down *iff* the mass of moldy lemons is sufficiently large. The intuition comes from noticing that the first agent to exit the market is the best type. When the best type does not trade, although the market price increases because the average quality of the remaining pool is worse, each agent's *marginal rate of substitution* between quantity and price weakly *increases*. Therefore, all remaining agents' incentives to trade actively in the market are at least as strong as they previously were. Therefore, the only way to generate a large cascade of exits defined by types beyond the best type exiting the market is if the initial entry mass of moldy lemons is sufficiently large, a result reminiscent of Akerlof (1970) and Azevedo and Gottlieb (2017) under exclusive contracts.

We then turn to understand the more relevant question: Under what circumstances can a normal functioning market collapse due to a *small* mass of moldy lemons entering. We extend the model by endowing agents with an outside option to market participation. These outside options may represent payoffs obtainable to agents outside of the market.[3] Alternatively, they can represent fixed participation costs. For example, an agent may enjoy a higher utility by staying out of the market, because entering the market may take time, effort, and cost. Moreover, it could capture the time and effort required to search for a counterparty or supplier, the effort and monetary cost of negotiating and verifying contract terms, or paying other brokerage or settlement fees.[4]

The second result stated in Theorem 2 states that in the presence of outside options, a small mass of moldy lemons can generate a large cascade of exits and market shut downs. The intuition comes from the fact that outside options provide agents reservation utility against which the utility from trade in the market is compared. Hence, the marginal utility characterization of the baseline model is replaced by a total utility representation. Using total utility, the exit of the best type of agent has negative spillover effects on the remaining agents, as the closing of one market lowers the quantity available at lower price to all remaining agents and reduces overall utility. Moreover, all remaining market trades require higher market clearing prices, which makes the outside option more appealing despite each agent's high marginal rate of substitution. In this case, the exit of a single good type can cause the

[2] Though not derived in this paper, a strategic foundation of this allocation is the discriminatory ascending-price auction in Attar et al. (2021). See subsection 2.3 for more details.

[3] For example, an outside option is an agent's reservation value in search and matching models.

[4] This interpretation is similar to entry fees studied by Bisin and Gottardi (1999, 2003).

next best agent to exit the market because the relative value of their outside option increases. Therefore, further exits can happen and remaining agents may suffer additional utility loss and so on. An important property of our exit cascades Theorem is that it only requires the *first marginal agent's* outside option to be knife-edged and similar to their potential market utility. Exit cascades occur even as the size of each remaining agent's outside option *declines* relative to their market utility.

Our last analytical result—(Proposition 3)—shows an important comparative static relating the degree of asymmetric information to exit cascades. Economies with more uncertainty regarding the underlying types of the agents are less vulnerable to exit cascades than economies that feature more known types. More specifically, economies that feature what we call more *coarse partitions* of types—weighted averages of multiple types—can be interpreted as having more uncertainty about the true type of each agent in the partition. Economies with more coarse partitions are less likely to feature exit cascades for a given mass of moldy lemons. The reason is that relative weight of each partition is larger when multiple times comprise it than the weight of each type alone. Thus, a mass of mold lemons sufficient to generate an exit cascade in an economy with fully disaggregated types may be insufficient to generate the same amount of exit with a more coarse partition. For example, the cost increase due to moldy lemons that triggers a cascade, in which agent 1 exiting causes agent 2 to exit, may not be sufficient to cause both agents 1 and 2 to exit as a group when agent 2 exits only because of the spillover effect associated with agent 1's exit. Therefore, economies in which there is more precise information or certainty about each agent's type are more likely to feature exit cascades. This result provides a theoretical underpinning to the information production models of Gorton and Ordoñez (2019); Dang et al. (2020); Gorton and Ordoñez (2020) in which market failure arise endogenously due to the incentives to produce private information.

A numerical example confirms the formal results of Theorems 1 and 2 and our comparative static result of Proposition 3. The example also shows that markets are more prone to shut down as the number of types in the economy grow large and each type is more similar. In particular, if the masses of adjacent types are combined into a single type with the same average probability of a bad state as the original partition, then the introduction of moldy lemons is less likely to cause a cascade of exits. Therefore, markets are more vulnerable to shutdowns when there are many different types, even when aggregate risk or uncertainty remain the same. The reason is due to Proposition 3: the sufficient mass of new moldy lemons needed to trigger market shutdowns increases as the relative mass of existing agents increases. In other words, if both the best type and the second best type do not know whether they are the best type or not, the initial trigger of cascade by the best type is less likely to occur.

Overall, our results suggest a parsimonious yet realistic way of generating sudden market shutdowns without imposing additional structure or institutional details on the model. Thus, our model is widely applicable to many different markets and contexts, and provides simple insights on the properties of market shutdowns.

The paper proceeds as follows: Section 2 presents the non-exclusive contracting framework of Attar et al. (2021), and introduces the critical concept of entry-proofness. Section 3 introduces moldy lemons and states Theorem 1. Section 4 introduces outside options, states Theorem 2 and Proposition 3, and discusses policy implications. Section 5 provides a numerical example and simulations to show the impact of moldy lemons and outside options in the non-exclusive contracting environment.

Relation to the Literature. Recent work by Attar et al. (2011, 2014, 2021) characterizes the terms and efficiency of trade in markets with nonexclusive competition. Attar et al. (2011) show, in a special case with linear utility and capacity constraints, that markets with nonexclusive competition cease to function for high quality agents alá Akerlof (1970) and always remain open for the lowest quality agents.[5] However, the market will not completely unravel as agents become more risky due to the linear and continuous nature of the model. Attar et al. (2014, 2021) consider general preferences and non-linear pricing, but do not establish a market unraveling result outside of the possibility of not finding a competitive equilibrium as in the Rothschild and Stiglitz (1976). In fact, a deterioration of type distribution might rather increase the likelihood of positive trades by bad types instead of triggering a complete market shutdown (which is pooling with zero quantities). Thus, an Akerlof-esque market unraveling result has not been established in a non-exclusive contracting environment with general preferences and non-linear pricing. The goal of this paper is to fill this gap in the literature and derive general principles under which small changes in underlying fundamentals, which we coin the entry of moldy lemons, cause markets subject to adverse selection with non-exclusive contracting to unravel and shutdown.

Our paper relates to the literature studying non-exclusive contracting in economies with adverse selection pioneered by (e.g. Pauly, 1974; Jaynes, 1978; Hellwig, 1988; Glosten, 1994). Allocations in these non-exclusive contracting environments are recursive; agents trade in layers consisting of multiple contracts.[6] More recent results extend to generalized settings with divisible goods, general preferences, and multiple types (e.g Attar et al., 2011, 2014,

[5] This is true aside from the trivial case where any buyer's expected valuation of the good is lower than the lower bound of the valuation of sellers.

[6] Bisin and Gottardi (1999, 2003) show that some form of non-linear pricing is needed to make compatible price taking behavior with asymmetric information.

2021; Dubey and Geanakoplos, 2019). Among these, our paper builds directly upon Attar et al. (2021) and Dubey and Geanakoplos (2019) but with a different focus; we study the conditions under which small changes in the distribution of agents causes markets to unravel.

Our focus on the unraveling of competitive equilibrium due to adverse selection dates back to Akerlof (1970) and Rothschild and Stiglitz (1976), and recently generalized by Hendren (2013, 2014) and Azevedo and Gottlieb (2017). In the classic example, Akerlof (1970) shows that markets unravel when the cost to serve the market exceeds the marginal willingness of each agent to trade. More recently, the basic but powerful insight has been embedded into macro and investment models to explain how information frictions can amplify business cycles, (e.g. Kurlat, 2013), and how long-term asset markets can suffer self-fulfilling liquidity freezes (e.g. Malherbe, 2014; Asriyan et al., 2019). Unlike these papers, our model features a richer non-exclusive contracting environment in which agents are not restricted to trade at a single market clearing price. We show that with non-exclusive contracts, active markets are robust to small changes in underlying fundamentals and Akerlof unraveling does not generally occur. Hendren (2014) shows that equilibrium in insurance economies with exclusive contracting must unravel either via Akerlof-pricing or failure to satisfy competitive-Nash equilibrium à la Rothschild-Stigliz when the distribution of types either contains a continuous interval, or the type whose accident probability equals 1. We show that this result is partially sensitive to competitive contracting as in Azevedo and Gottlieb (2017) and the exclusive contracting assumption. In general, entry of a type with accident probability equal to 1 is not sufficient to cause markets to unravel under non-exclusive contracting.

Recent applications of non-exclusive contracting under adverse selection have been applied to security design (e.g. Asriyan and Vanasco, 2021), and asset markets with heterogeneously informed buyers (e.g. Kurlat, 2016). One common feature of non-exclusivity is that neither security nor asset markets more broadly are fully separating; there is always some form of cross-subsidization among markets. Our model inherits semi-pooling in equilibrium, but our focus is on how small changes in underlying fundamentals cause markets to unravel. Auster et al. (2021) study a form of non-exclusivity in search with adverse selection. Workers can apply to as many jobs as possible, but ultimately sell their labor to a single firm. Fully separating equilibria are precluded because high types always send some applications to low-wage-offering firms to hedge against remaining unemployed.

Philippon and Skreta (2012) show that the failure of the price mechanism and market unraveling justify public interventions during liquidity or credit freezes. A key insight in their framework is that interventions impact the set of agents that choose to participate in government programs, which in turn impacts trade in the market. In a nonexclusive contracting framework, policies that increase entry cost prevent market unraveling only if the policy can discriminate among types; otherwise, a uniform cost increase makes markets more prone to unraveling because the best types exit first, which raises the cost of trade for all remaining types.

2 Model

The model builds on Attar et al. (2021). We briefly lay out the specifics of the environment and state the relevant theorems in Attar et al. (2021) that aid in our analysis of the conditions under which markets shut down as worse types enter the market—*i.e., moldy lemons*.[7]

The demand side of the market consists of a finite number of privately informed agents indexed by $i \in I \equiv \{1, .., n\}$ with a strictly positive measure of each type, m_i. Utility for each type is given by $u_i(q, t)$ and assumed to be continuous, quasi-concave in the arguments and strictly decreasing in t. Generically, q represents the quantity of a good that is consumed and t is the transfer required to obtain the good. An important feature of the model is that privately informed types are ordered by a single-crossing property (Milgrom and Shannon, 1994), $\forall i < j,\ q < q',\ t,\ t'$:

$$u_i(q, t) \leq u_i(q', t') \Rightarrow u_j(q, t) < u_j(q', t').$$

Single-crossing implies that a higher type is at least as willing as a lower type to trade an additional unit of the good for an additional transfer.[8] More generally, we can define a marginal rate of substitution without assuming differentiable utility functions. Let $\tau_i(q, t)$ be the supremum set of prices, p, such that

$$\tau_i(q, t) \equiv \sup \left\{ p : u_i(q, t) < \max_{q' \geq 0} u_i(q + q', t + pq') \right\}.$$

Hence, $\tau_i(q, t)$ is the slope of the indifference curve at an additional quantity, $q' > q$, and can be considered as a (pseudo-)marginal rate of substitution for agent i at consumption bundle (q, t). An important assumption for our analysis of "moldy lemons" to come is that, absent a transfer, a strictly positive endowment of q lowers agents' marginal rate of substitution:

Assumption 1 $\tau_i(q, 0) \leq \tau_i(0, 0)$, $\forall i, q > 0$.[9]

[7] We refer the reader to their paper for detailed proofs.

[8] Here we are using strict single-crossing condition rather than weak single-crossing, which allows for equality. This is because we want to focus on the strict notion of market breakdown in light of Corollary 1 in Attar et al. (2021).

[9] The same assumption is also used in Attar et al. (2021) in their analysis.

By way of concrete examples, the model translates into the insurance economy of Rothschild and Stiglitz (1976), where i indicates an agent's probability of loss, q is the amount of insurance purchased, and t is the insurance premium. In a credit economy, i indexes borrower default probability, q is the loan quantity demanded, and t is the gross loan promise made to the lender.

Contracts, defined by the pair (q, t) for $q \geq 0$, are supplied by competitive risk-neutral, expected profit maximizers. The suppliers possess a linear production technology with a unit cost, $c_i > 0$, which is increasing in i. Adverse selection occurs if c_i is increasing in type with the single-crossing condition. That is, higher types wish to trade more than low types, but the cost of servicing these types is higher. Define by $\bar{c}_i$ the expected unit cost of serving $j \geq i$ (the upper-tail conditional expected cost) given that higher types will be willing to trade any contract offered to type i. Formally,

$$\bar{c}_i \equiv E[c_j | j \geq i] = \frac{\sum\limits_{j \geq i} m_j c_j}{\sum\limits_{j \geq i} m_j}. \tag{1}$$

2.1 The Concept of Entry-proof, Inactive Markets

An inactive market cannot be an equilibrium if an entrant can propose a set of contracts that leads to positive trades with nonnegative profits. In other words, an inactive market must be resilient to entry and "entry-proof" as proposed by Attar et al. (2021). We first describe when an inactive market is entry-proof and then describe when active markets are entry-proof.

The key is to first consider the no-trade contract, $(0, 0)$, as any agent's outside option. Then, a market is entry proof iff, for any menu of contracts an entrant offers, the buyer's best response earns the entrant zero expected profit. That is, the entry-proof condition is given by

$$\textbf{Condition EP}: \qquad \tau_i(0, 0) \leq \bar{c}_i \qquad \forall i. \tag{2}$$

Condition EP simply says that there will be no trade in a market when the cost of offering a contract for agent i exceeds type i's marginal utility of not trading, given that all other types higher than i must also be served.[10] Theorem 1 in Attar et al. (2021) states that **Condition EP** is necessary and sufficient for markets to be inactive.

[10]We will use contracts and markets interchangeably. We want to think about all contracts with potentially different price-quantity pairs as individual markets following Bisin and Gottardi (1999); Dubey and Geanakoplos (2002); Dubey et al. (2005).

We sketch the proof here since the arguments are useful in our extensions. The single-crossing condition implies that an entrant offering an arbitrary menu of contracts will end up trading (q_i, t_i) with type i and $q_j \geq q_i$ with all agents $j \geq i$. The expected profit to the entrant of this menu is $\sum_i m_i [t_i - c_i q_i]$. Using summation by parts, the expected profit can be written in terms of layers $(q_i - q_{i-1})$ and $(t_i - t_{i-1})$: $\sum_i \left(\sum_{j \geq i} m_j\right) [t_i - t_{i-1} - \bar{c}_i(q_i - q_{i-1})]$, where $(q_0, t_0) \equiv (0, 0)$. Moreover, it must be the case that agent i is willing to trade the additional layer on top of the original set of contracts that yielded (q_{i-1}, t_{i-1}), if the entrant's offer is accepted. That is, the marginal rate of substitution of agent i at the original allocation times the new layer must exceed its cost: $\tau_i(q_{i-1}, t_{i-1})(q_i - q_{i-1}) > t_i - t_{i-1}$. In addition, the entrant cannot make a loss on each type given the expected cost, $\bar{c}_i$, to serve all types $j > i$ that will also accept the contract. Therefore, $t_i - t_{i-1} - \bar{c}_i(q_i - q_{i-1}) \geq 0$. Single-crossing implies that $q_i \geq q_{i-1}$, so combining the two previous inequalities, entry will be non-profitable when $\tau_i(q_{i-1}, t_{i-1}) \leq \bar{c}_i$. Then, using the fact that type $i-1$ prefers their optimal trade to no trade, we have $\tau_{i-1}(q_{i-1}, t_{i-1}) < \tau_{i-1}(0, 0)$, and so does type i. Invoking the assumption that, absent transfers, agents' marginal rate of substitution is weakly decreasing in quantities, $\tau_i(\underline{q}_i, 0) \leq \tau_i(0, 0)$, where $\underline{q}_i \in [0, q_{i-1}]$ is the quantity that makes agent i indifferent between (q_{i-1}, t_{i-1}) and $(\underline{q}_i, 0)$. Finally, by quasi-concavity of preferences, $\tau_i(q_{i-1}, t_{i-1}) \leq \tau_i(\underline{q}_i, 0)$, we have the desired result $\tau_i(q_{i-1}, t_{i-1}) \leq \tau_i(\underline{q}_i, 0) \leq \tau_i(0, 0) \leq \bar{c}_i$.

We consider a notion of market breakdown proposed by Attar et al. (2021) where any menu of contracts that strictly attracts at least some agents yields a strictly negative expected profit, even if the buyer's best response is most favorable to the entrant. A corollary to Theorem 1 is that this notion of market breakdown occurs if and only if **Condition EP** is satisfied when the preferences are strictly convex and strict single-crossing holds. Hence, our notion of market shutdown is equivalent to **Condition EP**.

2.2 The Concept of Entry-proof, Active Markets

The main question we are after is, under what conditions does the entry of worse type buyers (or a worsening of the buyer type distribution) cause active markets to shut down? In order to answer this question, we first ask when active markets are entry-proof in the sense that entry of a supplier is unprofitable. Armed with the answer to the latter question based on Attar et al. (2021), we can answer the former.

Trade in the market is non-exclusive in the sense that no agent can be stricken from trading with multiple firms or suppliers. Therefore, we must define the market tariff, or the minimum aggregate transfer that is made across active markets to obtain aggregate consumption, q. Define the aggregate market tariff by $T(q)$. We will assume that $T(q)$

is convex and the domain is a compact interval with lower bound equal to 0. Then, all agents choose q_i to maximize $u_i(q_i, T(q_i))$. An allocation, $(q_i, T(q_i))_{i \in I}$ is *implemented* by the market tariff, T, if $q_i = \arg\max_q u_i(q, T(q))$. This allocation is *budget feasible* if suppliers make non-negative expected profits at the market tariff

$$\sum_i m_i \left[T(q_i) - c_i q_i\right] \geq 0. \tag{3}$$

Under exclusive contracting environment, an entrant only needs to consider the agent's direct utility functions. However, under non-exclusive contracting environment, an agent can combine any menu of potential new contracts with trades along the existing market tariff, T. Thus, an active market is entry proof if an entrant cannot make positive expected profits given agents' ability to combine contracts. Therefore, the entrant faces types with indirect utility functions of trading a proposed new contract (q', t') in addition to the existing allocation $(q, T(q))$:

$$u_i^T(q', t') \equiv \max\left\{u_i(q + q', T(q) + t' : q)\right\} \tag{4}$$

From the entrant's perspective, an agent's individual rationality constraint is determined by the indirect utility of not trading the proposed contract on top of the existing market tariff, $u_i^T(0, 0)$. We can define the marginal rate of substitution along the indirect utility functions as above by $\tau_i^T(q', t')$.[11] As shown by Attar et al. (2021), the indirect utility functions will also satisfy single-crossing because the primitives satisfying the same condition.

Attar et al. (2021) proposes an additional assumption, which is slightly strong than Assumption 1, implying that the marginal rates of substitution are nonincreasing in q for any type i and transfer t:

Assumption 2 *For all i and t, $\tau_i(q, t)$ is nonincreasing in q.*

Intuitively, this assumption implies that a higher quantity always reduces each agent's willingness to pay for any additional quantity. For the given market tariff, T, and the allocation, $(q_i, T(q_i))$, we can define $\tau_i^T(0, 0)$ as the supremum of the set of prices p for the indirect utility function, $u_i^T(0, 0)$—that is,

$$u_i(q_i, T(q_i)) = u_i^T(0, 0) < \max\left\{u_i^T(q', pq') : q'\right\} = \max\left\{u_i(q + q', T(q) + pq' : q, q')\right\}.$$

[11]This is possible because: 1) the maximizers in (4) are continuous from Berge's Maximization Theorem; 2) the market tariff, T, is convex; 3) the utility functions, $u_i(q, t)$, are weakly quasi-concave in (q, t) and strictly decreasing in t; 4) and, hence, the indirect utility functions u_i^T are weakly quasi-concave in (q, t) and strictly decreasing in t.

With this, we can invoke the necessity and sufficiency result of **Condition EP** to state that a market tariff is entry-proof iff:

$$\forall\, i, \quad \tau_i^T(0,0) \leq \bar{c}_i. \tag{5}$$

This condition states that an active market is entry-proof if and only if the cost required to enter the market exceeds the willingness of each agent to trade the contract on top of the allocation they may already obtain. For each agent, the utility they receive from their market trades must be at least as large as trading the proposed new contract given the cost required to serve the market:

$$\text{for each } i, \quad u_i(q_i, T(q_i)) \geq \max\left\{u_i(q+q', T(q) + \bar{c}_i q' : q, q')\right\}. \tag{6}$$

From the entrant's perspective, by convention of letting $q_0 \equiv 0$, setting $q' = q_i - q$, and applying condition (6) for each layer, $q \in [q_{i-1}, q_i]$, we see that the implied market tariff necessary to induce entry must be at least as large as the pre-entry tariff:

$$\text{for each } i \text{ and } q \in [q_{i-1}, q_i], \quad T(q_i) \leq T(q) + \bar{c}_i(q_i - q). \tag{7}$$

For a given $q = q_{i-1}$, we have $T(q_i) \leq T(q_{i-1}) + \bar{c}_i(q_i - q_{i-1})$. Using the budget-feasibility condition from (3), and re-writing it in terms of layers, we have

$$\sum_i \left(\sum_{j \geq i} m_j\right) \left[T(q_i) - T(q_{i-1}) - \bar{c}_i(q_i - q_{i-1})\right] \geq 0. \tag{8}$$

Therefore, combining (7) and (8) implies

$$T(q_i) = T(q_{i-1}) + \bar{c}_i(q_i - q_{i-1}). \tag{9}$$

It must also be true that the allocation, $u_i(q_i, T(q_i))$, implied by the new layer, $q_i - q_{i-1}$ maximizes the utility of all the agents that choose it, given that the new market tariff must rise to serve all agents. Hence, for each i,

$$u_i(q_i, T(q_i)) = \max\left\{u_i(q_{i-1} + q', T(q_{i-1}) + \bar{c}_i q' : q')\right\}. \tag{10}$$

Finally, because the tariff is convex and satisfies (7) and (10), it must be affine with slope $\bar{c}_i$ over the interval $[q_{i-1}, q_i]$.

With this, Attar et al. (2021) state **Theorem 2**—An allocation $(q_i, T(q_i))_{i \in I}$ is budget-feasible and implemented by an entry-proof convex market tariff, T, with domain $[0, q_n]$ if and only if

1. $(q_0, T(q_0)) \equiv (0, 0)$,
2. $q_i - q_{i-1} \in \arg\max \{u_i(q_{i-1} + q', T(q_{i-1}) + \bar{c}_i q' : q')\}$ for each i,
3. $q_{i-1} < q_i \Rightarrow T$ is affine with slope $\bar{c}_i$ over $[q_i, q_{i-1}]$ for each i.

To sum up, an affine convex market tariff is entry proof as long as the upper-tail conditional cost of entry exceeds the marginal willingness of all agents to trade the additional layer on top of their pre-entry allocation. This market tariff consists of layers of trade with unit prices $\bar{c}_i$ that trace out a polygon with an upward kink at each $q_{i+1} \geq q_i$ for each $i \in I$.

2.3 Discussion of Strategic Foundations

We are taking the strategic foundations of the allocations from **Theorem 2** in Attar et al. (2021) state above as given. There are two ways to micro-found this allocation. The first is the discriminatory ascending-price auction used in Attar et al. (2021). In particular, each time a new price is quoted, each seller publicly announces the maximum quantity he stands ready to trade at this price. Once this auctioning phase is completed, the buyer decides which quantities to purchase from which sellers in a nonexclusive way. The second is the notion of competitive-pooling in general equilibrium pioneered by Dubey and Geanakoplos (2002) and extended to a non-exclusive contracting by Dubey and Geanakoplos (2019).

One final issue with the entry-proof market tariff of Attar et al. (2021) worth noting is uniqueness. The convex market tariff is unique if the solution to each agent's maximization problem is unique. This is guaranteed if the agents' preferences are strictly convex, which we assume to be the case. The problem of multiplicity arises only if the marginal rate of substitution of some type i equals $\bar{c}_i$ over a whole interval of quantities, which is not a generic phenomenon (Attar et al., 2021).

3 Moldy Lemons

We now ask what happens to trades and the market tariff as increasingly worse types of agents enter the economy. What we have in mind are situations for which firms anticipate a very high cost of serving some types of agents who may generate losses with near certainty. For example, the COVID-19 pandemic led to a surge in defaults. Our notion of *market shutdown* is equivalent to the comparative static in which active markets become inactive in equilibrium, hence no trade.

To fix ideas, assume that a new agent of type $n+1$ (a "moldy lemon") enters the economy with mass m_{n+1}. The cost to serve this agent is $c_{n+1} > c_n$. Given that the unit cost of serving agent $n+1$ is strictly greater than the unit cost of serving any other agent, the upper-tail conditional expected cost of serving all agents must also rise. Specifically, the new upper-tail conditional expected cost for any agent i is given by

$$\tilde{\bar{c}}_i \equiv \frac{\sum_{j \geq i} m_j c_j + m_{n+1} c_{n+1}}{\sum_{j \geq i} m_j + m_{n+1}}, \tag{11}$$

and $\tilde{\bar{c}}_i > \bar{c}_i$. Then, it is clear that any market that was inactive *ex ante* agent $n+1$ enters remains inactive *ex post* because **Condition EP** implies $\tau_i(0,0) \leq \bar{c}_i < \tilde{\bar{c}}_i$. Intuitively, making the average quality of the pool worse will never lead to the opening of new markets.

Next, does the arrival of agent $n+1$ lead any active markets to become inactive and shut down? Recall that an active-market is subject to entry if the marginal willingness to trade for all agents is greater than the upper-tail conditional cost to serve them. Then, if a market was active and $\tau_i(0,0) > \tilde{\bar{c}}_i$ and $\tilde{q}_i > \tilde{q}_{i-1}$ for some i, the market will remain active with a new equilibrium level of aggregate trade given by quantities $(\tilde{q}_i)_{i \in N}$. Intuitively, active markets must be characterized by terms of trade that do not make agents worse off compared with not trading. If there is at least one type of agent that is lower than type $n+1$ willing to trade in a market given that the cost will rise, then the market will not close despite the presence of type $n+1$. Moreover, type $n+1$ causes the slope of the market tariff to rise along all segments of the polygon that trace out the new market tariff, $\tilde{T}(\tilde{q})$. Finally, active markets cease to remain active only when $\tau_i^{\tilde{T}}(0,0) \leq \tilde{\bar{c}}_i$, and $\tilde{q}_i > \tilde{q}_{i-1} \ \forall i$. From (11), moldy lemons lead to market breakdowns only when their mass, m_{n+1} is sufficiently large. Formally, we have the following:

Theorem 1 (Moldy Lemons)
Suppose the economy is populated with additional type $n+1$ (moldy lemons) with mass m_{n+1} and $c_{n+1} > c_n$. Then, for the new equilibrium affine aggregate tariff, $\tilde{T}$, and new quantities, $(\tilde{q}_i)_{i \in N \cup \{n+1\}}$, the following hold:

1. *All inactive markets (market shutdown) remain inactive under the new equilibrium.*

2. *All active markets will remain active and have the new steeper equilibrium slope for the aggregate tariff, $\tilde{T}$, as $\tilde{\bar{c}}_i > \bar{c}_i$ over $[\tilde{q}_{i-1}, \tilde{q}_i]$ for every i, if $\tilde{q}_i > \tilde{q}_{i-1}$.*

3. *Active markets become inactive and shut down iff $\tau_i^{\tilde{T}}(0,0) \leq \tilde{\bar{c}}_i$ and $\tilde{q}_i = \tilde{q}_{i-1}$ for each i.*

Proof. Statements 1 and 3 are trivial as explained in the discussion before the theorem. We show that statement 2 holds.

First, consider type 1. Suppose that agent 1 was trading a positive quantity $q_1 > 0$, without loss of generality. Suppose that type 1 agent exits the market after the entry of moldy lemons. Then, the quantity demanded trivially decreases as $\tilde{q}_1 = 0 < q_1$, and the market tariff increases as $\tilde{T}(q) \geq \tilde{\bar{c}}_2 q > \bar{c}_2 q > \bar{c}_1 q$ for any q in $[0, q_1]$ and $[q_1, q_2]$, regardless of the new trading quantities $\tilde{q}_2, \tilde{q}_3, \ldots, \tilde{q}_n$. Suppose that type 1 agent still trades a positive quantity in the market. The market tariff trivially increases to $\tilde{T}(q) = \tilde{\bar{c}}_1 q$ for any q in $[0, \tilde{q}_1]$ as in the previous case. Given the new optimal quantity for type 1, $\tilde{q}_1$, type 2 agent will face a higher slope of the affine tariff for the quantity $\tilde{q}_2 \geq \tilde{q}_1$ as

$$\frac{\tilde{T}(\tilde{q}_2) - \tilde{T}(\tilde{q}_1)}{\tilde{q}_2 - \tilde{q}_1} \geq \frac{\tilde{\bar{c}}_1 \tilde{q}_1 + \tilde{\bar{c}}_2(\tilde{q}_2 - \tilde{q}_1) - \tilde{\bar{c}}_1 \tilde{q}_1}{\tilde{q}_2 - \tilde{q}_1} = \tilde{\bar{c}}_2 > \bar{c}_2 = \frac{T(q_2) - T(q_1)}{q_2 - q_1},$$

because $\tilde{\bar{c}}_j > \bar{c}_j$ for any j.

Now consider an arbitrary type $i > 2$. Following the previous argument results in

$$\begin{aligned} \frac{\tilde{T}(\tilde{q}_i) - \tilde{T}(\tilde{q}_{i-1})}{\tilde{q}_i - \tilde{q}_{i-1}} &\geq \frac{\sum_{j<i} \tilde{\bar{c}}_j \left(\tilde{q}_j - \tilde{q}_{j-1}\right) + \tilde{\bar{c}}_i(\tilde{q}_i - \tilde{q}_{i-1}) - \sum_{j<i} \tilde{\bar{c}}_j \left(\tilde{q}_j - \tilde{q}_{j-1}\right)}{\tilde{q}_i - \tilde{q}_{i-1}} = \tilde{\bar{c}}_i \\ &> \bar{c}_i = \frac{T(q_i) - T(q_{i-1})}{q_i - q_{i-1}}, \end{aligned}$$

with the convention $\tilde{q}_0 \equiv 0$. Thus, the new slope for the affine market tariff $\tilde{T}$ becomes steeper in each and every interval of the new optimal quantity layers. ■

Theorem 1 states general properties of active and inactive markets under the new equilibrium. One important implication is that active markets will completely shut down only if $\tau_i(0, 0) \leq \tilde{\bar{c}}_i$ for all i. Hence, the mass of moldy lemons, m_{n+1}, should be large enough to generate market shutdowns.[12] Our market shutdown result extends the Akerlof-like unraveling result established in the non-exclusive contracting setting of Attar et al. (2011) to more general preferences and non-linear pricing. Their paper shows that even in the strategic setting of Rothschild and Stiglitz (1976), equilibrium always exists and is unique with non-exclusive competition. Hence markets fail in the Akerlof sense when the unique equilibrium is the no-trade equilibrium. More surprising, Theorem 1 above shows that no trade occurs in non-exclusive settings precisely when the separating equilibrium of Rothschild and Stiglitz (1976) exists.

Our next result addresses which layers are most susceptible to agents exiting. The single-

[12] Azevedo and Gottlieb (2017) show a similar result under an exclusive contracting environment. Hence, we are extending their results to non-exclusive contracting environment.

crossing condition implies that higher types have higher willingness to trade at higher prices. Hence, as worse types enter the market and increase the market tariff along each layer, the lowest types exit the market first. This is easiest to see for the case where the first type $i = 1$ is just indifferent to trading given the upper-tail conditional cost of serving all greater types $j > 1$ where $\tau_i(0,0) = \bar{c}_i < \tilde{\bar{c}}_i$, while other agents $j > 1$ could still have $\tau_j^{\tilde{T}}(0,0) > \tilde{\bar{c}}_j$. In this case, type 1 is no longer willing to trade at the new market tariff, $\tilde{T}(\tilde{q})$, given the additional cost required to make the tariff budget-feasible while all types $j > 1$ remain active in the market. Hence, only the first layer of trade, q_1, becomes inactive.

Proposition 1 *In equilibrium, the lowest type (cost) agents exit first as m_{n+1} increases.*

Proof. First, the marginal rate of substitution, $\tau_i(q,t)$, is nonincreasing in q. Second, utility, $u_i(q,t)$, is continuous and decreasing in t. Combining this with strict single-crossing, we have $u_{i-1}(0,0) \le u_{i-1}(\tilde{q}_{i-1}, \tilde{T}(\tilde{q}_{i-1})) \Rightarrow u_i(0,0) < u_i(\tilde{q}_{i-1}, \tilde{T}(\tilde{q}_{i-1}))$. By continuity, there exists m_{n+1}^i such that $\tilde{T}$ makes $u_{i-1}(0,0) > \max_q u_{i-1}(q, \tilde{T}(q))$ and $u_i(0,0) < \max_q u_i(q, \tilde{T}(q))$. Again by single-crossing, if $i-1$ exits, then any type $j < i-1$ also exits. ■

An important implication of Proposition 1 is that the arrival of moldy lemons (worse than worst types) has only a marginal negative effect on active markets. The reason is that when the first type drops out and the first layer of trade, $(q_1, \tilde{T}(q_1))$, is removed from the market, the marginal value of each remaining layer for each agent goes up relative to the alternative of no trade at $(0,0)$. In other words, $\tau_i(0, \tilde{T}(0)) \ge \tau_i(q_1, \tilde{T}(q_1))$. Hence, the spillovers from agents exiting *increase* rather than decrease the incentives for all remaining agents to trade. This force keeps the remaining markets active. On the one hand, **Condition EP** is very robust in the sense that a complete unraveling with non-exclusive contracting requires a tautological extremely large mass of bad types. On the other hand, there is a gap between the model and the real world phenomena that exhibit a sudden collapse of the markets after hitting the tipping point of the severity of adverse selection (Beltran et al., 2017; Calomiris and Gorton, 1991; Covitz et al., 2013; Mishkin, 1999; Ivashina and Scharfstein, 2010; Foley-Fisher et al., 2020).

4 Outside Options and Market Shutdown

We now show that a cascade of exits—a sequential shut down of multiple markets—is possible upon entry of a small mass of moldy lemons when outside options are present. An outside option may be thought of as an alternative to entering the market, which requires either a contractual barrier or a cost of entry. For example, signing a contract or searching for the right supplier may require significant time and effort. Alternatively, it may represent an external market into which agents can enter and secure a reservation utility. The important point is that agents compare outside utility to their utility from market trades.

Denote the utility from outside options (and not entering the market) as γ_i for type i. Type i now compares the set of market contracts to their outside option. Thus, the individual rationality condition for i becomes

$$V_i(T) = \max\left\{\gamma_i, \max_{q\geq 0} u_i(q, T(q))\right\} \tag{12}$$

for a given aggregate market tariff T.[13]

In general, there are two ways in which asymmetric information generates market unraveling. The first is Akerlof (1970) unraveling where trade breaks down and the zero-trade point is the unique equilibrium. The second notion of market unraveling relates to the non-existence of competitive equilibrium studied by Rothschild and Stiglitz (1976). It turns out that both notions of market unraveling are possible in our framework depending on the distribution of the value of outside options across agents. The following sub-sections present each case in turn.

4.1 Akerlof Unraveling: Cascade of Exits

In this subsection, we present our main result that a sequential market shutdown results from a cascade of exits. The main mechanism for the market shutdown is based on the spillover effects of exits across agents in the presence of outside options. We show that the form of spillovers needed to generate a market shutdown do not exist in the model without outside options.

We assume the following condition on the distribution of outside option values:

$$u_i(q,t) \geq \gamma_i \Rightarrow u_j(q,t) > \gamma_j \qquad \forall i < j,\ \forall q > 0,\ \forall t. \tag{13}$$

This condition implies that if agent i prefers a contract (q, t) with a positive quantity to the outside option, then agent $j > i$ also prefers the contract to the outside option, which is in line with the idea of single-crossing property. We discuss the role and micro-foundation of this assumption in subsection 4.3.

Agents who enter the market optimize their utility for the given aggregate market tariff just as in the baseline model without outside options. Therefore, the arguments in Theorem 2 of Attar et al. (2021) for budget feasible, entry-proof market tariffs also hold with outside options. In particular, the aggregate entry-proof convex market tariff is T with the slope

[13] Note outside options are irrelevant for equilibrium when they provide utility less than no trade.

of $\bar{c}_i$ for each layer $[q_{i-1}, q_i]$ for each i with the convention of $q_0 = 0$. Therefore, the same arguments used in statements 1 and 2 in Theorem 1 and Proposition 1 hold here, and we can say the following.

Proposition 2 *In any active market equilibrium, there is a cutoff-type $\theta \in N \cup \{0\}$ such that any agents with type less than or equal to θ exit the market and any agents with type greater than θ remain in the market.*

Proposition 2 follows from Proposition 1, as the lowest type agents exit first in any active market equilibrium. Proposition 2 will be important for eliminating any possible nonexistence of equilibrium discussed in the following subsection 4.2.

We now use Proposition 2 to discuss how exits of lowest type agents can generate a further cascade of exits. Agents decide to enter/remain in the market by comparing the total utility they achieve through market trades with the value of their outside option. Therefore, agents exit the market when outside options offer more utility than market trades. Because the lowest types exit first, all remaining agents face a higher aggregate market tariff, which lowers the total utility of the remaining higher types. Different from the baseline model without outside options, all remaining agents compare their *total utility* from market trades with their outside options. By contrast, without outside options, remaining agents only compare their *marginal rate of substitution* at no-trade with the aggregate market tariff as in equation (2). Therefore, the existence of outside options can create a cascade of exits and generate a larger decline of quantities traded compared with the baseline model without outside options.

We now formalize the cascade of exit argument. Suppose, without loss of generality, that at least agent 1 exits the market after the entry of moldy lemons, which occurs when **Condition IT** holds:

$$\max_{q \geq 0} u_1(q, \tilde{\bar{c}}_1 q) \leq \gamma_1. \tag{14}$$

Note that we do not need to check **Condition EP**, $\tau_1(0,0) \leq \tilde{\bar{c}}_1$, because each agent prefers the outside option over the no-trade contract as $\gamma > u_i(0,0)$ for any i. We can extend this intuitive condition to a condition that any agent with type j that is below i exits the market. We denote such a condition, **Condition ML(i)–Moldy Lemons**, as:

$$\max_{q \geq 0} u_j\left(q, \tilde{\bar{c}}_j q\right) \leq \gamma_j, \qquad \forall j < i, \tag{15}$$

where agents up to type i exit the market. **Condition ML(i)** is stricter than **Condition EP** because there is an additional case that prevents entry. Therefore, agents who did not exit the market under **Condition EP** may exit under **Condition ML**(i). We formally show that **Condition ML**(i) is necessary and sufficient to generate a cascade of exits up to agent i.

Theorem 2 (Cascade of Exits) *Any agent $j < i$ exits the market in equilibrium iff* ***Condition ML***(i) *is satisfied.*

Proof. The proof of sufficiency is straightforward as **Condition ML**(i) prevents entry of agents with type $j < i$ for the entry-proof market tariffs.

Now consider the proof of necessity. By Proposition 2, we check the lowest agent's entry decision for each candidate equilibrium. Consider an equilibrium in which agent 1 also enters the market. Then, the corresponding aggregate market tariff will be

$$T^0(q) \equiv \sum_{i \in N} \tilde{\tilde{c}}_i(q - q_{i-1}) \mathbf{1}\left\{q \in [q_{i-1}, q_i]\right\},$$

which is based on the same quantities $\{q_i\}_{i \in N}$ as in the equilibrium before the moldy lemons entered, where $\mathbf{1}\{\cdot\}$ is an indicator function. Under **Condition IT**, (14), type 1 agent exits the market and the updated aggregate market tariff is

$$T^1(q) \equiv \sum_{i \in N} \tilde{\tilde{c}}_i(q - q^1_{i-1}) \mathbf{1}\left\{q \in [q^1_{i-1}, q^1_i]\right\},$$

where $q^1_i - q^1_{i-1} \in \arg\max \left\{u_i(q^1_{i-1} + q, T^1(q^1_{i-1}) + \tilde{\tilde{c}}_i q : q\right\}$ with $q^1_1 = 0$. The exit of agent 1 lowers the utility of each agent because the first layer over the interval $[0, q_1]$ with the lowest tariff, $\tilde{\tilde{c}}_1$, disappears. Also, by the same arguments in the proof of Theorem 1, all the remaining agents are participating in the market with the higher average cost of service due to moldy lemons. Thus, agents $i > 2$ trading the next available lowest cost interval, $[q_1, q_2]$, suffer further utility declines even without the exit of agent 2. If the new aggregate market tariff, T^1, induces agent 2 to exit, then it must be the case that

$$\max_{q \geq 0} u_2\left(q, T^1(q)\right) = \max_{q \geq 0} u_2\left(q, \tilde{\tilde{c}}_2 q\right) \leq \gamma_2,$$

and exiting the market gives higher utility to agent 2.

Now extend the argument recursively to finish the proof by mathematical induction. For an arbitrary $k < i$, suppose that under any candidate equilibrium, agents up to k exit the market. Then, the new market tariff becomes

$$T^k(q) \equiv \sum_{i \in N} \tilde{\tilde{c}}_i(q - q^k_{i-1}) \mathbf{1}\left\{q \in [q^k_{i-1}, q^k_i]\right\},$$

where $q_i^k - q_{i-1}^k \in \arg\max \left\{u_i(q_{i-1}^k + q, T^k(q_{i-1}^k) + \tilde{\tilde{c}}_i q : q\right\}$ with $q_1^k = \cdots = q_k^k = 0$. If agent $k+1$ exits the market under T^k, then

$$\max_{q \geq 0} u_{k+1}\left(q, T^k(q)\right) = \max_{q \geq 0} u_{k+1}\left(q, \tilde{\tilde{c}}_{k+1} q\right) \leq \gamma_{k+1}$$

should hold, because agent $k+1$ will still enter the market otherwise. If the above inequality does not hold, then the equilibrium of active markets is determined starting from $q_{k+1} > 0$ and the initial assumption of $q_{k+1} = 0$ is violated. Therefore, **Condition ML**(i) is necessary. ■

Outside options have two roles. First, they generate discontinuous jumps in market quantities, q_i, after the entry of moldy lemons. A small mass of moldy lemons m_{n+1} will trigger type i agent to exit with strictly positive quantity q_i when i's utility is close to the utility from the outside option as $u_i(q_i, T(q_i)) \approx \gamma_i$. By contrast, quantities always change smoothly without outside options. Second, outside options generate negative spillovers that can trigger discontinuous exist cascades.[14]

We now consider how the distribution of types impacts the exit cascade result in Theorem 2. Specifically, how does the coarseness of agent types impact exit cascades? Let $\{I, m, u, c, \gamma\}$ represent an economy where $m = \{m_i\}_{i \in I}$, $u = \{u_i\}_{i \in I}$, $c = \{c_i\}_{i \in I}$, and $\gamma = \{\gamma_i\}_{i \in I}$. Consider another economy, $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$, that is a *coarser partition* of $\{I, m, u, c, \gamma\}$ if the following holds:

1. $\hat{I} \subset I$.
2. If $i \in \hat{I}$ and $i+1 \in \hat{I}$, then $\hat{m}_i = m_i$, $\hat{u}_i = u_i$, and $\hat{c}_i = c_i$.
3. If $i \in \hat{I}$ and $i+1, \ldots, i+k \notin \hat{I}$, while $i+k+1 \in \hat{I}$ or $i+k+1 > n$, where $k \geq 1$, then agent $i \in \hat{I}$ *includes* agents $i, i+1, \ldots, i+k \in I$ and $\hat{m}_i = \sum_{l=0}^k m_{i+l}$, $\hat{u}_i(q,t) = \dfrac{\sum_{l=0}^k m_{i+l} u_{i+l}(q,t)}{\sum_{l=0}^k m_{i+l}}$, $\hat{c}_i = \dfrac{\sum_{l=0}^k m_{i+l} c_{i+l}}{\sum_{l=0}^k m_{i+l}}$, and $\hat{\gamma}_i = \dfrac{\sum_{l=0}^k m_{i+l} \gamma_{i+l}}{\sum_{l=0}^k m_{i+l}}$.

The above definition implies that a coarser partition of an economy groups adjacent types of agents into one type. The mass of the new type of agent is equal to the sum of all masses for each type in the group, and the servicing cost and outside option values are the weighted average cost and outside option values, respectively. The utility of this new agent is the weighted average utility across different types. For example, for $I = \{1, 2, 3, 4, 5\}$ and

[14] This mechanism is similar to the worsening adverse selection after a price increase in Stiglitz and Weiss (1981).

$\hat{I} = \{1, 2, 4\}$, $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$ is a coarser partition of $\{I, m, u, c, \gamma\}$, if

$$\begin{aligned}
\hat{m} &= \{m_1, m_2 + m_3, m_4 + m_5\} \\
\hat{u} &= \left\{u_1, \frac{m_2 u_2 + m_3 u_3}{m_2 + m_3}, \frac{m_4 u_4 + m_5 u_5}{m_4 + m_5}\right\} \\
\hat{c} &= \left\{c_1, \frac{m_2 c_2 + m_3 c_3}{m_2 + m_3}, \frac{m_4 c_4 + m_5 c_5}{m_4 + m_5}\right\} \\
\hat{\gamma} &= \left\{\gamma_1, \frac{m_2 \gamma_2 + m_3 \gamma_3}{m_2 + m_3}, \frac{m_4 \gamma_4 + m_5 \gamma_5}{m_4 + m_5}\right\}.
\end{aligned}$$

Note that the upper-tail conditional expected cost remains the same.

Proposition 3 *Let $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$ be a coarser partition of $\{I, m, u, c, \gamma\}$ with $i \in \hat{I}$ and $i + 1 \notin \hat{I}$. Then, there exists a moldy lemon mass, m_{n+1}, such that i does not exit in $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$, while i exits in $\{I, m, u, c, \gamma\}$. Furthermore, if $i \in \hat{I}$ exits in $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$, then $i \in I$ exits in $\{I, m, u, c, \gamma\}$.*

Proof. First, we show that if the entry of a mass of moldy lemons m_{n+1} causes the new type $i \in \hat{I}$ to exit in the coarser partition $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$, then agent i exits in the original economy $\{I, m, u, c, \gamma\}$. Suppose that $i \in \hat{I}$, $i + 1, \ldots, i + k \notin \hat{I}$, and $i + k + 1 \in \hat{I}$, i exits the market in the economy $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$. Thus,

$$\max_{q \geq 0} \left[\frac{m_i u_i(q, \bar{\bar{c}}_i q) + \cdots + m_{i+k} u_{i+k}(q, \bar{\bar{c}}_i q)}{m_i + \cdots + m_{i+k}}\right] \leq \hat{\gamma} = \frac{m_i \gamma_i + \cdots + m_{i+k} \gamma_{i+k}}{m_i + \cdots + m_{i+k}}$$

holds. Suppose the contrary that i does not exit the market in the economy $\{I, m, u, c, \gamma\}$. Then,

$$\max_{q \geq 0} u_i(q, \bar{\bar{c}}_i q) = u_i(q_i, \bar{\bar{c}}_i q_i) > \gamma_i.$$

Thus, for each l, $u_{i+l}(q_i, \bar{\bar{c}}_i q_i) > \gamma_{i+l}$ holds by (13), where $1 \leq l \leq k$. However, this implies the weighted average of utilities would exceed the weighted average of outside options. In other words,

$$\max_{q \geq 0} \left[\frac{m_i u_i(q, \bar{\bar{c}}_i q) + \cdots + m_{i+k} u_{i+k}(q, \bar{\bar{c}}_i q)}{m_i + \cdots + m_{i+k}}\right] \geq \frac{m_i u_i(q_i, \bar{\bar{c}}_i q_i) + \cdots + m_{i+k} u_{i+k}(q_i, \bar{\bar{c}}_i q_i)}{m_i + \cdots + m_{i+k}} > \hat{\gamma},$$

which is a contradiction.

Now we show the converse that agent i, who exits the market in the economy $\{I, m, u, c, \gamma\}$ due to the entry of moldy lemons with mass m_{n+1}, may not exit in the coarser partition

$\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$ with the same mass of moldy lemons entering the market. Suppose that $i \in \hat{I}$, $i+1, i+2, \ldots, i+k \notin \hat{I}$, and $i+k+1 \in \hat{I}$. Agent i exits the market, if

$$\max_{q \geq 0} u_i(q, \tilde{\tilde{c}}_i q) \leq \gamma_i. \tag{16}$$

By continuity of the utility function, maximand (by Berge's maximum theorem), and upper-tail conditional expected cost, the left-hand side of (16) is continuously decreasing in the moldy lemon mass m_{n+1}. Then, there exists $\underline{m}$ such that when $m_{n+1} = \underline{m}$,

$$\max_{q \geq 0} u_i(q, \tilde{\tilde{c}}_i q) = u_i(q_i, \tilde{\tilde{c}}_i q_i) = \gamma_i.$$

For each l, $u_{i+l}(q_i, \tilde{\tilde{c}}_i q_i) > \gamma_{i+l}$ holds by (13), where $l \geq 1$. Again, the utility of $i \in \hat{I}$, the weighted average of utilities, become

$$\begin{aligned} \max_{q \geq 0} \left[\frac{m_i u_i(q, \tilde{\tilde{c}}_i q) + \cdots + m_{i+k} u_{i+k}(q, \tilde{\tilde{c}}_i q)}{m_i + \cdots + m_{i+k}} \right] &\geq \frac{m_i u_i(q_i, \tilde{\tilde{c}}_i q_i) + \cdots + m_{i+k} u_{i+k}(q_i, \tilde{\tilde{c}}_i q_i)}{m_i + \cdots + m_{i+k}} \\ &> \hat{\gamma} = \frac{m_i \gamma_i + \cdots + m_{i+k} \gamma_{i+k}}{m_i + \cdots + m_{i+k}}, \end{aligned}$$

and type $i \in \hat{I}$ agent does not exit. Hence, there exists a moldy lemon mass $m_{n+1} = \underline{m}$, which makes i to exit in $\{I, m, u, c, \gamma\}$ but not in $\{\hat{I}, \hat{m}, \hat{u}, \hat{c}, \hat{\gamma}\}$. ∎

This result shows that markets are less vulnerable to shutdowns and exit cascades when the partition of types in the economy becomes coarser. At the other extreme, exit cascades are more likely when the distribution of types is close to a continuum. The reason is that the sufficient mass of new moldy lemons needed to trigger market shutdowns increases as the relative mass of exiting agents increases.

A corollary of Proposition 3 is that markets can be more vulnerable to exits as the number of types in the economy grows large despite no change in underlying aggregate risk or uncertainty. The result emphasizes why having multiple types in the model is important for generating large swings in trade with a small mass of moldy lemons. Thus, a model with only two types, while tractable and perhaps sufficient to highlight certain forces, does not correctly capture the vulnerability of the market to exit cascades more generally.

We believe this result is apt to describe the recent CLO-market freezes. CLOs are purchased by an increasing variety of agents, such as banks, insurance companies, private equity funds, mutual funds, etc. Proposition 3 suggests that these markets are more likely to dry up with only a small change in fundamentals, as more agents become prominent players in the market.

4.2 Rothschild-Stiglitz Unraveling

In this subsection we focus on the case in which equilibrium does not exist—that is, Rothschild and Stiglitz (1976) unraveling—similar to the nonexistence of equilibrium in Attar et al. (2014). A necessary condition for the non-existence of equilibrium is that assumption (13) does not hold.[15] Everything else in the model remains the same.

Suppose the moldy lemons, type $n+1$ with $c_{n+1} > c_n$, enter the market. Without loss of generality, we assume that all agents enter the market in the equilibrium before the moldy lemons joined. As in the previous section, the new upper-tail conditional expected cost is given by (11). Denote equilibrium quantities by $\tilde{q}_i$ for each i that enters the market where

$$\tilde{q}_i - \tilde{q}_{i-1} \in \arg\max_q \left\{ u_i \left(\tilde{q}_{i-1} + q, \tilde{T}(\tilde{q}_{i-1}) + \tilde{\bar{c}}_i q \right) : q \right\},$$

and $\tilde{T}(q)$ is the equilibrium aggregate affine tariff with slope $\tilde{\bar{c}}_i$ over $[\tilde{q}_{i-1}, \tilde{q}_i]$. In addition, set $\tilde{q}_0 = \tilde{q}_j = 0$ for any j who exits the market.

In this case, agent entry and exit decisions in equilibrium may be either strategic complements or substitutes à la Bulow et al. (1985). Suppose that type i agent does not enter the market in equilibrium, and for simplicity, suppose that all other agents do enter the market. The rest of the arguments go through even if there are other agents who exit the market. Since i exits the market, $\tilde{q}_i = 0$ and

$$\tilde{T}(\tilde{q}_{i+1}) - \tilde{T}(\tilde{q}_{i-1}) = \tilde{\bar{c}}_{i+1} \left(\tilde{q}_{i+1} - \tilde{q}_{i-1} \right).$$

Now compare that to a hypothetical equilibrium with the same quantities $(\tilde{q}_j)_{j<i}$ and corresponding market tariff $\tilde{T}$ in which type i enters the market. Hence, $\tilde{q}_i > \tilde{q}_{i-1}$ and

$$\tilde{T}(\tilde{q}_{i+1}) - \tilde{T}(\tilde{q}_{i-1}) = \tilde{\bar{c}}_{i+1} \left(\tilde{q}_{i+1} - \tilde{q}_i \right) + \tilde{\bar{c}}_i \left(\tilde{q}_i - \tilde{q}_{i-1} \right) < \tilde{\bar{c}}_{i+1} \left(\tilde{q}_{i+1} - \tilde{q}_{i-1} \right),$$

implying that $i+1$ will have higher utility under this quantities and market tariff. By the same logic that utilities are increasing when lower types are present applies to all agents greater than i. Because their utilities in the market increase, agents with type greater than i are more likely to enter the market when i enters the market. Therefore, an entry of a lower type agent generates one-way strategic complementarity to higher type agents.

However, the effect can go the other way for agent j with type lower than i, i.e. $j < i$. If

[15] Theorem 1 of Attar et al. (2014) shows that an equilibrium exists only if the adverse selection problem is severe enough, implying that different types' incentives are not too closely aligned.

the upper-tail conditional expected cost of agent j excluding i is less than c_i—that is,

$$\frac{\tilde{T}(\tilde{q}_j) - \tilde{T}(\tilde{q}_{j-1})}{\tilde{q}_j - \tilde{q}_{j-1}} = \frac{\sum_{k\geq j, k\neq i} m_k c_k}{\sum_{k\geq j, k\neq i} m_k} < c_i,$$

then the entry of i will increase the upper-tail conditional expected cost for agents with type j and lower, as the new upper-tail conditional expected cost is

$$\frac{\sum_{k\geq j, k\neq i} m_k}{\sum_{k\geq j} m_k} \frac{\sum_{k\geq j, k\neq i} m_k c_k}{\sum_{k\geq j, k\neq i} m_k} + \frac{m_i}{\sum_{k\geq j} m_k} c_i,$$

which is a convex combination of $\frac{\tilde{T}(\tilde{q}_j) - \tilde{T}(\tilde{q}_{j-1})}{\tilde{q}_j - \tilde{q}_{j-1}}$ and c_i, so

$$\frac{\tilde{T}(\tilde{q}_j) - \tilde{T}(\tilde{q}_{j-1})}{\tilde{q}_j - \tilde{q}_{j-1}} = \frac{\sum_{k\geq j, k\neq i} m_k c_k}{\sum_{k\geq j, k\neq i} m_k} < \frac{\sum_{k\geq j, k\neq i} m_k}{\sum_{k\geq j} m_k} \frac{\sum_{k\geq j, k\neq i} m_k c_k}{\sum_{k\geq j, k\neq i} m_k} + \frac{m_i}{\sum_{k\geq j} m_k} c_i.$$

Because of this cost increase, agent j's maximum utility in the market decreases, and j may exit the market because of it. Hence, an entry of a higher type agent can generate one-way strategic substitutability to a lower type agent. Therefore, the overall effect of the equilibrium remains ambiguous.

Because of the potential strategic substitutability, there can be no equilibrium stemming from the interaction between strategic complements and strategic substitutes, which is similar to how an equilibrium breaks down in Rothschild and Stiglitz (1976). For a simpler example, suppose that type n agent exits the market because the aggregate market tariff T is too high compared to n's outside option, i.e. $\max_q u_n(q, T(q)) \leq \gamma_n$. However, the exit of n decreases T to $T^\dagger$, which may make n want to enter again. Then, the entry of n will change (increase) $T^\dagger$ to T again, which will make n want to exit again. Furthermore, the entry or exit could affect other agents $i < n$ in the market to exit or enter the market, complicating the equilibrium existence even more by increasing or decreasing T even further. Thus, the outside options generate key interactions between entry/exit decisions of different agents that can destabilize the otherwise very robust and stable entry-proof equilibrium of Attar et al. (2021).

However, depending on distributional assumptions, outside options also generate the cascade of exits from a small change in fundamentals as in subsection 4.1. Specifically, condition (13) generates Proposition 2, which is important because an exit of the lowest (best) type agent always has an unambiguous, negative, and strategic effect to higher type agents—exits of the lower type agents generate additional exits. Therefore, an additional assumption that provides structure to the distribution of the outside option values is needed to guarantee that the equilibrium exists.

4.3 Discussion of Outside Options

Though sufficient, condition (13) is not necessary for exits to generate additional exits due to negative spillovers. However, without some structure, the additional complexity of Rothschild and Stiglitz (1976)-type of non-existence problems arise. In addition, the pattern of exits could become more complicated as there might be no cutoff type, θ, that partitions agents into those who exit and remain. In particular, some intermediate valued agent may remain in the market if the agent's outside option utility is very low.

We introduce two different interpretations of outside options to justify assumption (13). The first interpretation is a fixed entry cost. Suppose that each agent must pay a fixed entry cost given by $\xi > 0$ if they choose to enter the market. Upon entry, the agent trades a positive quantity, q, while paying the market tariff of $T(q)$. The outside option is not paying the entry cost of ξ, or a negative payment with zero trade quantity: $u_i(0, -\xi)$. Then, by continuity and Assumption 2, there exists $\underline{q}_i$ such that

$$u_i(\underline{q}_i, 0) = u_i(0, -\xi) = \gamma_i$$

for any i. Also, by the single-crossing property,

$$u_i(\underline{q}_i, 0) \geq u_i(0, -\xi) \Rightarrow u_j(\underline{q}_i, 0) > u_j(0, -\xi),$$

for any $j > i$. Therefore, $\underline{q}_i$ is decreasing in the index i, and

$$u_i(q, t) \geq \gamma_i \Rightarrow u_j(q, t) > \gamma_j, \qquad \forall i < j,$$

by the single-crossing property. Hence, (13) is a result of the single-crossing condition.

The second way to interpret outside options is to consider the opportunity cost of agents entering a separate market that requires costly verification of agent's type.[16] In this market, agents pay a fixed cost of κ to trade, and the market can verify each agents type. Therefore, agent 1 may be happy to pay κ and get the lowest tariff c_1 for the quantity q_1, whereas agent n would not be happy to pay κ and pay the highest tariff c_n. Thus, whenever agent $j > i$ exits, agent i may also exit.

[16]The secondary market structure of agency mortgage-backed securities (MBS) is a good example. A majority of MBS are traded in the to-be-announced (TBA) market, which pools heterogeneous MBS into a few liquid TBA contracts but induces adverse selection. At the same time, traders can trade high-value MBS outside the TBA market in a much less liquid specified-pool (SP) market by specifying the individual CUSIP, but traders pay higher trading cost in the SP market (Huh and Kim, 2021).

In the model, exit cascades require some specific parametric conditions. The most restrictive condition is that the outside option value for agent 1 should be very close to the ex ante equilibrium utility level $\max_{q\geq 0} u_1(q, T(q)) \simeq \gamma_1$. However, once agent 1 exits due to the entry of moldy lemons, agent 2's maximum utility may experience a downward jump. Therefore, γ_2 can be well below the ex ante equilibrium utility level, $\max_{q\geq 0} u_2(q, T(q)) > \gamma_2$, and agent 2 would still exit the market. Moreover, the downward jumps in utility are cumulative as agent n will face all the decreases in utility from the exits of previous agents, $1, 2, \ldots, n-1$. If we consider a model where the set of eligible participants is endogenous (e.g., agent 1 is the marginal type who is close to being indifferent between entering and exiting the market), the condition that agent 1's outside option utility level be sufficiently close to their ex ante equilibrium utility level arises naturally. Therefore, the required parametric conditions may not be as restrictive as they appear.

Finally, one can think of a general equilibrium model in which the utility levels of outside options are endogenously affected by entry/exit decisions of all agents.[17] Such general equilibrium interactions are out of the scope of this paper but will be an interesting direction for future research. Nevertheless, our exit cascade results continue to hold even if the value of outside options declines as the best types exit the market. More specifically, exit cascades occur as long as the downward jump in market utility for remaining agents is larger than the decline in value of the outside option.

4.4 Implications and Broader Discussion

The model of moldy lemons with outside options generates amplification effects—a small mass of moldy lemons can generate a sudden market shutdown. This property resembles real-world financial market movements and has an important policy implication: relatively inexpensive policy interventions can prevent sudden and costly market collapses. Policies need only to prevent the small mass of moldy lemons from contaminating the market. For example, if the social planner lowers the market tariff with a total subsidy of $(\tilde{\bar{c}}_1 - \bar{c}_1)q_1 \sum_{i\in I} m_i$, then it is sufficient to prevent type 1 agent's exit and the subsequent exit cascade. This policy is desirable as long as the potential welfare losses, $\sum_{i\in I}(u_i(q_i, T(q_i)) - \gamma_i)$[18], on top of

[17]The current formulation of outside options under the two interpretations discussed above does not allow the exit decisions of agents to impact the value of others' outside options. For example, the fixed entry fee is independent of other agents' entry and exit decisions. Also, if the outside option "market" can identify the true type of each agent, entry of moldy lemons or other agents into this outside option market will not affect the utility of the lower type agents.

[18]Since suppliers are competitive, they break even in any equilibrium even under complete breakdown. Therefore, the measure of social welfare is simply the sum of utilities across all types of agents (buyers).

the spillovers to other markets is higher than the intervention cost.[19] Therefore, our model provides a simple yet important reason to support market functioning *even with adverse selection* to prevent more widespread market breakdowns.

The cascade of exits is determined by both the degree of adverse selection and agents' outside options. If entry is very costly—for example, because of high entry or regulatory costs due to heavy usage of balance sheets—the outside option of agents not entering the market could be more profitable. Then, there will be more exits in the market. A moderate reduction of such costs (or reduction of the opportunity cost) could drastically change the allocation by preventing the chain of exits and sudden collapse of the quantities traded in the equilibrium.

Moreover, our results highlight the importance of monitoring the *best (most reliable/least costly) agent* in the market. Even though monitoring the overall condition and the worst (most risky/most costly) participants would be crucial in identifying the effect of moldy lemons, exits of the most reliable agents are the trigger of the exit cascade. Hence, monitoring the exit incentives of the best participant in each market is important for predicting and preventing shut downs ex ante.

The model does not rely on detailed market structure or other types of complex interactions of the agents. Therefore, the model could be applied to various contexts and markets with adverse selection to provide insights on how adverse selection problems can cause partial or full market shutdowns through a variety of changes within a given setting.

Our result that economies with more types (less coarse partitions) are more vulnerable to exit cascades provides a general theoretical underpinning to the information production literature (see for example, Gorton and Ordoñez (2019, 2020); Dang et al. (2020)). In particular, one interpretation is that economies with more types grouped together have more imprecise or opaque information about each individual type. This interpretation is apt for models where an agent's type is stochastic as in production economies or asset holdings where agents maximize over the expected value of their types. Our model shows that if more precise information about each agent's type results in more recognizable agents in the economy—a less coarse partition—and more asymmetric information between buyers and sellers, then the market is more unstable. This is in line with Dang et al. (2020) who show that information production can lead to a collapse in the market. Our result shows a similar phenomenon with a more simple, static model.

[19] There are many complicated issues related to optimal interventions such as changing incentives under the new rules (or mechanisms) as discussed in Philippon and Skreta (2012).

5 Numerical Example

This section provides a numerical example to highlight the main analytical results. In particular, consider the canonical insurance market with binary loss model of Rothschild and Stiglitz (1976) and Hendren (2014) following the idea of Dubey and Geanakoplos (2019).[20] We first compare the results between the baseline model of Attar et al. (2021) with and without outside options. Then, we consider the comparative statics on the partition of types.

5.1 Model Setup and Equilibrium

Agents have initial endowment $(e, 0)$ for (e_g, e_b), where e_g and e_b represent good state and bad state endowment, respectively. They will consume $(x_g, x_b) = (e, 0)$ under autarky, where x_g and x_b represent good state and bad state consumption, respectively. Suppose that agents have constant relative risk aversion (CRRA) and agent i's utility function is

$$v_i(x_g, x_b) = p_i \log(1 + x_b) + (1 - p_i) \log(1 + x_g),$$

where p_i is the probability of agent i faces a loss and receives bad state endowment. The marginal rate of substitution for agent i is

$$\tau_i(x_g, x_b) \equiv \frac{\dfrac{\partial v_i(x_g, x_b)}{\partial x_b}}{\dfrac{\partial v_i(x_g, x_b)}{\partial x_g}} = \frac{p_i}{1 - p_i} \frac{1 + x_g}{1 + x_b}.$$

Following the assumption of competitive suppliers in the main model, the service cost for suppliers is $c_i = p_i$, so they need p_i amount of x_g to insure $1 - p_i$ amount of x_b to agent i. Under nonexclusive contracts, suppliers require the upper-tail conditional expected cost,

$$\bar{c}_i = \sum_{j \geq i} \frac{m_j p_j}{\sum_{j \geq i} m_j},$$

where m_i is the relative mass of agent i.

Denote the additional utility level of taking the outside option on top of utility under no trade, $v_i(e, 0)$, as γ for each i. Agents will compare the level of utility they can get from their optimal consumption bundle in the market to the level of utility they can get from the

[20]Note that the baseline model of Attar et al. (2021) and our extension with outside options in this paper are much more general and can be applied to many other contexts.

outside option and decide whether to enter or exit the market. Utilizing the results on the general model, the **Condition ML(i)** for this model is

$$v_i(x_g^i, x_b^i) \leq \gamma_i = v_i(e, 0) + \gamma,$$

where $v_i(e, 0) + \gamma$ is decreasing in i. If the above inequality holds, then any agent $j \leq i$ exits the market and we set $x_b^j = 0$.

5.2 Effect of Outside Options and Moldy Lemons

We verify the main result of this paper with the numerical model. Figure 3 shows equilibrium of the baseline model in the top panel and equilibrium of the model with outside options in the bottom panel. For each panel, the horizontal axis represents consumption in the good state, x_g, and the vertical axis represents consumption in the bad state, x_b. Each colored curve represents the consumption possibility frontier for a given mass of moldy lemons in the market. Different shapes of dots represent each agent's optimal consumption bundle given mass of moldy lemons and other agents' consumption in the equilibrium.

First, the results show that x_b decreases monotonically with the increase in the mass of moldy lemons. As more moldy lemons enter the market, the upper-tail conditional expected cost $\tilde{\bar{c}}_i$ increases, depressing the consumption possibility frontier for every agent in the market. With the higher cost, agents purchase less (insurance) contracts.[21]

Second, the results show a market shutdown in the model with outside options, which does not exist in the model without outside options. Even though total quantities decrease in both models, the model without outside options shows a smooth contraction of quantities traded as the mass of moldy lemons rises. Thus, even when the moldy lemons mass is 0.4, all agents still enter the market and trade in positive amounts. In contrast, the model with outside option shows the exit of agents and total market shutdown at the moldy lemons mass of 0.3. Thus, the numerical exercise shows how the existence of outside options can generate complete market shutdowns even for a small mass of moldy lemons.[22]

5.3 Coarse Partition of Types and Moldy Lemons

Using the model with outside options, we analyze comparative statics of Proposition 3 on the division of types depicted in Figure 4. In particular, we group adjacent types of agents

[21] This decrease in quantities may not always be the case in the general model, because the income effects might dominate the substitution effects. The functional form of this exercise precludes the case of insurance being a Giffen good.

[22] The definition of complete market shutdown here is the exit of all agents except for moldy lemons.

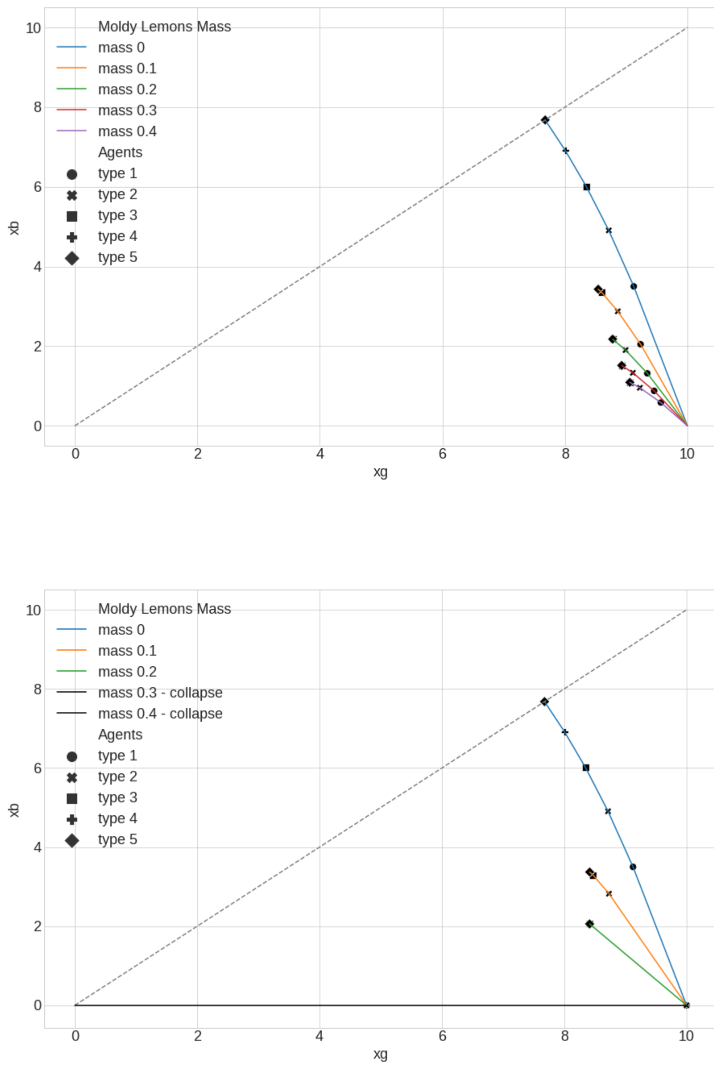

Figure 3: Consumption bundles of models without and with outside options

Note: Each curve represents consumption possibility frontier and consumption bundles of each agent represented by different shape of dot for a different mass of moldy lemons.

into a single type whose service cost is the weighted average of individual type's cost, and a mass equal to the sum of each type's mass. In particular, we combine type 1 and 2 together to create a new type 1 agent with probability and mass as

$$\hat{p}_1 = \frac{m_1 p_1 + m_2 p_2}{m_1 + m_2}$$
$$\hat{m}_1 = m_1 + m_2.$$

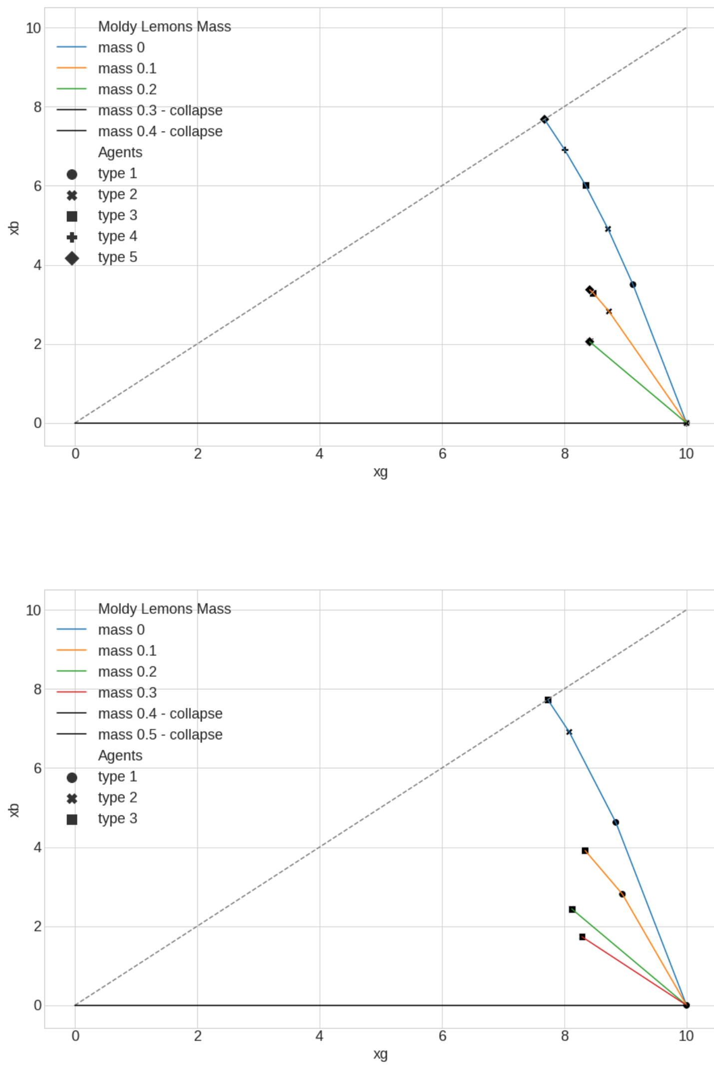

Figure 4: Consumption bundles of models with different partition of types

Note: Each curve represents consumption possibility frontier and consumption bundles of each agent represented by different shape of dot for a different mass of moldy lemons.

Similarly, we also combine type 3 and 4 in the baseline case into a new type 2 agent as

$$\hat{p}_2 = \frac{m_3 p_3 + m_4 p_4}{m_3 + m_4}$$
$$\hat{m}_2 = m_3 + m_4,$$

while simply renaming the previous agent 5 as agent 3. Therefore, the market-wide uncertainty and service cost remain the same as before.

The top panel of Figure 4 is the baseline case as shown in the bottom panel of Figure 3, while the bottom panel of Figure 4 is the case with a more coarse partition. The numerical results show that the economy with a coarser partition of types is less vulnerable to moldy lemons–there are fewer exits across varying masses of moldy lemons. Also, full market shutdown does not happen even when the moldy lemons mass is 0.3, which generated a full market collapse under the baseline case.

6 Conclusion

We show that the entry of a small mass of the worst type of agents (moldy lemons) can induce a cascade of exits and market shutdown under non-exclusive competition in the presence of outside options. Without outside options, entry of a small mass of lemons does not generate a cascade of exits and complete market shutdown because agents' marginal rates of substitution are (weakly) increasing. With outside options, the entry of moldy lemons causes trade quantities to discontinuously fall because the exit of an agent decreases total utility of the remaining agents, which could trigger a cascade of exits. Our results suggest a parsimonious yet realistic way of generating sudden market shutdowns without imposing additional structure, relying on belief- or sentiment-driven runs, or modeling institutional details. Thus, our model is widely applicable to many different markets and contexts.

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Appendix

A Details of Numerical Simulations

For computational tractability, we would like to represent agents' optimization problem as isomorphic optimization problem across all agents by adjusting the endowments e^i for each agent i. This can be done by exploiting the single-crossing property and any agent $i > j$ would consume as much as agent j does in equilibrium. For a given e^i, the optimal consumption bundle is

$$\begin{aligned} x_g^* &= (1-p_i)e^i - \frac{p_i - \bar{c}_i}{1-\bar{c}_i} \\ x_b^* &= \frac{p_i - \bar{c}_i}{\bar{c}_i} + \frac{(1-\bar{c}_i)p_i}{\bar{c}_i} e^i, \end{aligned}$$

for an interior solution. If it is a corner solution, then $(x_g^*, x_b^*) = \left(0, \frac{1-\bar{c}_i}{\bar{c}_i} e^i\right)$ or $(x_g^*, x_b^*) = (e^i, 0)$.

From the results in the general model, we know that agent 1 first decides on the optimal quantity q_1^* for the given price $\bar{c}_1$ and then agent 2 decides on q_2^* for the price $\bar{c}_2$ on top of purchasing q_1^*, and so forth. Agent 1's optimal consumption bundle is

$$\begin{aligned} x_g^1 &= (1-p_1)e - \frac{p_1 - \bar{c}_1}{1-\bar{c}_1} \\ x_b^1 &= \frac{p_1 - \bar{c}_1}{\bar{c}_1} + \frac{(1-\bar{c}_1)p_1}{\bar{c}_1} e, \end{aligned}$$

assuming that they have interior solutions without loss of generality. For agent 2, the same optimality condition should hold, but the budget constraint is different from the previous representation. This is because agent 2 can purchase the bundle in a cheaper price $\frac{\bar{c}_1}{1-\bar{c}_1}$ instead of $\frac{\bar{c}_2}{1-\bar{c}_2}$ up to x_b^1. Therefore, the updated budget constraint for agent 2 becomes

$$\begin{aligned} x_g^2 &= e - \frac{\bar{c}_1}{1-\bar{c}_1} x_b^1 - \frac{\bar{c}_2}{1-\bar{c}_2}(x_b^2 - x_b^1) \\ &= e + \left(\frac{\bar{c}_2}{1-\bar{c}_2} - \frac{\bar{c}_1}{1-\bar{c}_1}\right) x_b^1 - \frac{\bar{c}_2}{1-\bar{c}_2} x_b^2. \end{aligned}$$

Therefore, we simply change the endowment from e to

$$e^2 = e + \left(\frac{\bar{c}_2}{1-\bar{c}_2} - \frac{\bar{c}_1}{1-\bar{c}_1}\right) x_b^1$$

for the agent 2's budget constraint. Thus, the optimal consumption bundle for agent 2 is

$$x_g^2 = (1-p_2)e^2 - \frac{p_2 - \bar{c}_2}{1-\bar{c}_2}$$
$$x_b^2 = \frac{p_2 - \bar{c}_2}{\bar{c}_2} + \frac{(1-\bar{c}_2)p_2}{\bar{c}_2}e^2.$$

Agent 3's problem is isomorphic to agent 2's problem except that agent 3's endowment is

$$e^3 = e + \left(\frac{\bar{c}_2}{1-\bar{c}_2} - \frac{\bar{c}_1}{1-\bar{c}_1}\right)x_b^1 + \left(\frac{\bar{c}_3}{1-\bar{c}_3} - \frac{\bar{c}_2}{1-\bar{c}_2}\right)x_b^2,$$

and agent 3's optimal consumption bundle becomes

$$x_g^3 = (1-p_3)e^3 - \frac{p_3 - \bar{c}_3}{1-\bar{c}_3}$$
$$x_b^3 = \frac{p_3 - \bar{c}_3}{\bar{c}_3} + \frac{(1-\bar{c}_3)p_3}{\bar{c}_3}e^3.$$

For a general agent i, agent i's updated endowment is

$$e^i = e + \sum_{j<i}\left(\frac{\bar{c}_{j+1}}{1-\bar{c}_{j+1}} - \frac{\bar{c}_j}{1-\bar{c}_j}\right)x_b^j,$$

and agent i's optimal consumption bundle becomes

$$x_g^i = (1-p_i)e^i - \frac{p_i - \bar{c}_i}{1-\bar{c}_i}$$
$$x_b^i = \frac{p_i - \bar{c}_i}{\bar{c}_i} + \frac{(1-\bar{c}_i)p_i}{\bar{c}_i}e^i.$$

Given these setup, the parameters of the model are as in the following Table 1.

Parameter	Description	Value
e	good state endowment	10
$(p_1, p_2, \ldots, p_5)$	probability of bad state	$(0.1, 0.15, \ldots, 0.3)$
$(m_1, m_2, \ldots, m_5)$	mass of each type	$(0.2, 0.2, \ldots, 0.2)$
γ	outside option utility level	0.0758

Table 1: Parameter values for numerical simulations

For the numerical procedure, we check **Condition ML(i)** starting from $i = 1$ and updating the endowment of $e^{i+1} = e$ whenever the condition is satisfied. The algorithm repeats this until it finds the agent that does not violate the test. Then, we proceed with the rest of the agents' consumption quantities using the iterative representation.

The algorithm that solves this numerical model is the following: For each i,

1. Calculate the endowment of agent i, e^i, using the previous agents' quantities.

2. Derive the optimal quantity for agent i, (x_g^i, x_b^i) under e^i and $\bar{c}_i$.

3. Compute the utility level $v_i(x_g^i, x_b^i)$ and compare that to $v_i(e, 0) + \gamma$.

4. If it is above $v_i(e, 0) + \gamma$, then the (x_g^i, x_b^i) quantity is the optimal quantity. Otherwise, set $(x_g^i, x_b^i) = (e, 0)$.

5. Move to the next agent $i + 1$ and repeat from the first step.